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Fracture Mechanics Analysis by the Universal Stress Intensity Factors for Next Generation Aircraft & Spacecraft

E.G. Ladopoulos
Interpaper Research Organization
8, Dimaki Str.
Athens, GR - 106 72, Greece
eladopoulos@interpaper.org

Abstract

The theory of *"Relativistic Elasticity"* is investigated for the design of the new generation large aircraft with turbojet engines and speeds in the range of 50,000 km/h and for the new generation spacecraft of any speed. The new theory shows that there is a considerable difference between the absolute stress tensor and the stress tensor of the moving frame even in the range of speeds of 50,000 km/h. For much bigger speeds of the next generation spacecraft, like c/3, c/2, 3c/4, or 0.80c (c=speed of light), the difference between the two stress tensors is very much increased. Therefore, for the next generation spacecraft with very high speeds, then the relative stress tensor will be very much different than the absolute stress tensor and the relative stress intensity factors different than the absolute stress intensity factors. Furthermore, for velocities near the speed of light, then the values of the relative stress tensor are very much bigger than the corresponding values of the absolute stress tensor. The proposed theory of *"Relativistic Elasticity"* is a combination between the theories of "Classical Elasticity" and "Special Relativity" and results in the ***"Universal Equation of Elasticity"*** and in the ***"Universal Stress Intensity Factors"***. For the structural design of the new generation aircraft and spacecraft the stress tensor of the airframe will be used in combination to the singular integral equations method and the fracture mechanics behavior is analyzed and checked. This stress tensor is reduced to the solution of a multidimensional singular integral equation and for its numerical evaluation will be used the *Singular Integral Operators Method (S.I.O.M.).*

Key Word and Phrases

Relativistic Elasticity, Fracture Mechanics Analysis, Aircraft, Spacecraft, Relative Stress Tensor, Absolute Stress Tensor, Stationary and Moving Frames, Energy-Momentum Tensor, Multidimensional Singular Integral Equations, Singular Integral Operators Method (S.I.O.M.), Universal Equation of Elasticity, Universal Stress Intensity Factors.

1. Next Generation Aircraft and Spacecraft

The big evolution of the jet engines and the high performance axial – flow compressor have considerably increased the possibilities of turbomachines applied in aircrafts. The concern for very light weight in the aircraft propulsion application, and the desire to achieve the highest possible isentropic efficiency by minimizing parasitic losses, led inevitably to axial-flow compressors with cantilever airfoils of high aspect ratio. Also, the turbojet engines were found to experience severe vibration of the rotor blades at part speed operation. The increasing evolution of aeroelasticity in aircraft turbomachines continues to be under active investigation, driven by the needs of aircraft powerplant and turbine designers.

International Aeronautical Industries have as main scope to achieve a competitive technological advantage in certain strategic areas of new and rapidly developing advanced technologies, by which in the longer terms, can be achieved increased market share. Such a considerably big market share includes the design of a new generation large aircraft with speeds even in the range of 50,000 km/h. The future of such very fast aircraft is not too far. So, the application of new generation turbojet engines makes possible the design of such type of large aircraft and thus there is a need of elastic stress analysis and fracture mechanics analysis for the construction of the total parts of such type of next generation aircraft.

Also, the target of the International Space Agencies is to achieve in the future, next generation spacecraft moving with very high speeds, even approaching the speed of light. How far is this future ? According to the present research this future could be much closer than everybody believes. Over the next decades next generation spacecraft should be built. In such cases of the next generation spacecraft the relative stress tensor will be much different than the absolute stress tensor and so special solids should be used for the construction of such spacecraft. The type of the proper material for the construction of the next generation spacecraft is under investigation and will be very much different than the usual composite materials. Thus, a fracture mechanics analysis of the next generation spacecraft should be done.

By the present research it will be shown that there is a significant difference between the absolute stress tensor and the stress tensor of the airframe even in the range of speeds of 50,000 km/h. On the other hand, for bigger speeds the difference of the two stress tensors is very much increased. Thus, for bigger velocities like c/3, c/2, 3c/4 or 0.80c (c=speed of light) the relative stress tensor is very much different than the absolute one, while for velocities near the speed of light the values of the relative stress tensor are much bigger than the corresponding values of the absolute stress tensor. The study of the connection between the stress tensors of the absolute frame and the airframe is included in the theory proposed by E.G.Ladopoulos [30] - [32] under the term *"Relativistic Elasticity"* and the final formula which results from the above theory is called the ***"Universal Equation of Elasticity"***and the ***"Universal Stress Intensity Factors"***. Thus, by the present investigation the theory of *"Relativistic Elasticity"* will be applied for the elastic stress analysis design of the next generation aircraft and spacecraft.

In addition, E.G.Ladopoulos [1]-[16] and E.G.Ladopoulos et al. [17]-[22] proposed several linear singular integral equation methods applied to elasticity, plasticity and fracture mechanics applications. In the above studies the Singular Integral Operators Method (S.I.O.M.) is investigated for the numerical evaluation of the multidimensional singular integral equations in which is reduced the stress tensor analysis of the linear elastic theory. Furthermore, the theory of linear singular integral equations was extended to non-linear singular integral equations, too. [23]-[29]. The theory of *"Relativistic Elasticity"* will be applied to the design of the elastic stress analysis and fracture mechanics analysis of the airframes. *"Relativistic Elasticity"* is derived as a generalization of the classical theory of elastic stress analysis for stationary frames. Thus, for future aerospace applications the difference between the relative and the absolute stress tensors will be of increasing interest. Furthermore, the classical theory of elastic stress analysis began to be analyzed in the early nineteenth century and was further developed in the twentieth century. In the past were written several important monographs on the classical theory of elasticity. [33]-[52].

On the contrary, over the past years special attention has been concentrated on the theoretical aspects of the special theory of relativity. Thus, some classical monographs were written, dealing with the theoretical foundations and investigations of the special and the general theory of relativity. [53]–[60]. Also, a very important point which will be shown by the current investigation is that the **"*relative stress tensor is not symmetrical*"**, while, as it is well known, the **"*absolute stress tensor is symmetrical*"**. This difference is very important for the design of the next generation aircraft and spacecraft of very high speeds. So, the foundations of the theory of *"Relativistic Elasticity"* for airstructures lead to a general theory, in which no restriction is made with regard to the relative motion. This general theory is also reduced to one class of relative motion, uniform in direction and velocity.

2. Relativistic Elastic Stress Analysis for Airframes

The state of stress at a point in the stationary frame S^0, is defined by the symmetrical stress tensor given by the following formula: (Fig.1)

$$\sigma^0 = \begin{bmatrix} \sigma_{11}^0 & \sigma_{12}^0 & \sigma_{13}^0 \\ \sigma_{21}^0 & \sigma_{22}^0 & \sigma_{23}^0 \\ \sigma_{31}^0 & \sigma_{32}^0 & \sigma_{33}^0 \end{bmatrix} \tag{2.1}$$

where:
$$\sigma_{21}^0 = \sigma_{12}^0, \sigma_{31}^0 = \sigma_{13}^0, \sigma_{32}^0 = \sigma_{23}^0 \tag{2.2}$$

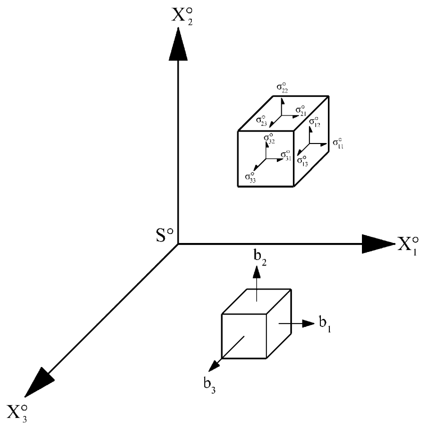

Fig. 1 The state of stress σ_{ik}^0 in the stationary system S^0

Consider further an infinitesimal face element df with a directed normal, defined by a unit vector **n**, at definite point p in the three-space of a Lorenz system. The matter on either side of this face element experiences a force which is proportional to df.

Thus, the force is valid as:

$$\mathrm{d}\boldsymbol{\sigma}(\mathbf{n}) = \boldsymbol{\sigma}(\mathbf{n})\,\mathrm{d}f \tag{2.3}$$

Beyond the above, the components $\sigma_i(\mathbf{n})$ of $\boldsymbol{\sigma}(\mathbf{n})$ are linear functions of the components n_k of **n**:

$$\sigma_i(\mathbf{n}) = \sigma_{ik} n_k, \; i,k = 1,2,3 \tag{2.4}$$

where σ_{ik} denotes the elastic stress tensor, which can be also called the relative stress tensor, in contrast to the space part σ_{ik}^0 of the total energy-momentum tensor T_{ik}, referred as the absolute stress tensor. [53], [54} (Fig. 2).

Furthermore, the connection between the absolute and relative stress tensors is defined as:

$$\sigma_{ik}^0 = \sigma_{ik} + g_i u_k, \; i,k = 1,2,3 \tag{2.5}$$

in which g_i are the components of the momentum density **g** and u_k the components of the velocity **u** of the matter.

The connection between **g** and the energy flux **s**, is valid as:

$$\mathbf{g} = \mathbf{s}/c^2 \tag{2.6}$$

where c denotes the speed of light (= 300.000 *km*/*sec*).

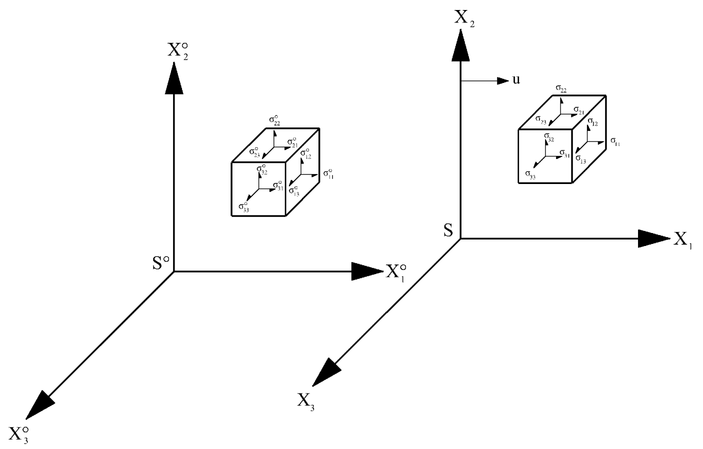

Fig. 2 The state of stress σ_{ik}^{0} in the stationary system S^{0} and σ_{ik} in the airframe system with velocity u parallel to the x_1 - axis.

On the contrary, the total work done per unit time by elastic forces on the matter inside the closed surface f is given by the following relation:

$$W = \int_f (\boldsymbol{\sigma}(\mathbf{n}) \cdot \mathbf{u}) \mathrm{d} f = \int_f \sigma_{ik} n_k u_i \, \mathrm{d} f = -\int_\upsilon \frac{\vartheta(u_i \sigma_{ik})}{\vartheta x_k} \mathrm{d}\upsilon \, , \, i,k = 1,2,3 \tag{2.7}$$

where the integration in the last integral is extended over the interior υ of the surface *f*.

So, the work done on an infinitesimal piece of matter of volume $\delta\upsilon$ is valid as:

$$\delta W = -\frac{\vartheta(u_i \sigma_{ik})}{\vartheta x_k} \delta\upsilon \tag{2.8}$$

Furthermore, (2.8) must be equal to the increase per unit time of the energy inside $\delta\upsilon$:

$$\frac{\mathrm{d}}{\mathrm{d}t}(h\delta\upsilon) = \delta W \tag{2.9}$$

in which h denotes the total energy density, including the elastic energy and $\mathrm{d}/\mathrm{d}t$ is the substantial time derivative.

Eq. (2.9) can be further written as:

$$\frac{\mathrm{d}}{\mathrm{d}t}(h\delta\upsilon) = \left(\frac{\vartheta h}{\vartheta t} + \frac{\vartheta h}{\vartheta x_k} u_k \right) \delta\upsilon + h\delta\upsilon \frac{\vartheta u_k}{\vartheta x_k} = \left[\frac{\vartheta h}{\vartheta t} + \frac{\vartheta}{\vartheta x_k}(h u_k) \right] \delta\upsilon \tag{2.10}$$

which finally leads to the relation:

$$\frac{\vartheta h}{\vartheta t} + \frac{\vartheta}{\vartheta x_k}(h u_k + u_i \sigma_{ik}) = 0 \tag{2.11}$$

Consequently, the total energy flow is given by the formula:

$$\mathbf{s} = \mathbf{hu} + (\mathbf{u} \cdot \boldsymbol{\sigma}) \tag{2.12}$$

where $(\mathbf{u} \cdot \boldsymbol{\sigma})$ denotes a space vector with components $(\mathbf{u} \cdot \boldsymbol{\sigma})_k = u_i \sigma_{ik}$.

So, the total momentum density can be written as:

$$\mathbf{g} = \frac{\mathbf{s}}{c^2} = \mu \mathbf{u} + \frac{(\mathbf{u} \cdot \boldsymbol{\sigma})}{c^2} \tag{2.13}$$

in which $\mu = h/c^2$ is the total mass density, including the mass of the elastic energy.

From (2.5) and (2.13) we obtain:

$$\sigma_{ik} - \sigma_{ki} = -g_i u_k + g_k u_i = [-(\mathbf{u} \cdot \boldsymbol{\sigma})_i u_k + (\mathbf{u} \cdot \boldsymbol{\sigma})_k u_i] / c^2 \neq 0 \tag{2.14}$$

which shows that the relative stress tensor is not symmetrical, in contrast to the absolute stress tensor (2.1) which is symmetrical.

In the stationary frame S^0 the velocity $u^0 = 0$ and thus, from (2.5), (2.12) and (2.13) one obtains the following expressions:

$$\sigma_{ik}^0 = \sigma_{ik} = \sigma_{ki} = \sigma_{ki}^0 \ (i, k = 1,2,3) \tag{2.15}$$

Furthermore, the mechanical energy-momentum tensor satisfies the following formula:

$$T_{ik} U_k = -h^0 U_i \tag{2.16}$$

where U_i denotes the four-velocity of the matter, in the Lorentz system and $U_i^0 = (0,0,0,ic)$.
Thus, the following scalar can be formed:

$$U_i T_{ik} U_k / c^2 = U_i^0 T_{ik}^0 U_k^0 / c^2 = -T_{44}^0 = h^0(x_1) \tag{2.17}$$

with $h^0(x_1)$ the invariant rest energy density considered as a scalar function of the coordinates (x_i) $(i = 1,2,3)$ in S. (Fig. 2)

Also, by applying the tensor:

$$\Delta_{ik} = \delta_{ik} + U_i U_k / c^2 \tag{2.18}$$

which satisfies the relations:

$$U_i \Delta_{ik} = \Delta_{ik} U_k = 0 \tag{2.19}$$

then, the following symmetrical tensor can be formed:

$$S_{ik} = \Delta_{i1} T_{1m} \Delta_{mk} = S_{ki} \tag{2.20}$$

which is orthogonal to U_i:

$$U_i S_{ik} = S_{ik} U_k = 0 \tag{2.21}$$

By combining eqs. (2.16), (2.17) and (2.20) one has:

$$S_{ik} = T_{ik} - h^0 U_i U_k / c^2 \tag{2.22}$$

Also, in the stationary system S_0 we have:

$$S_{ik}^0 = \sigma_{ik}^0 = \sigma_{ik} \,,\; S_{i4}^0 = S_{4i}^0 = 0 \tag{2.23}$$

Eq. (2.22) can be further written as:

$$T_{ik} = \xi_{ik} + S_{ik} \tag{2.24}$$

where:

$$\xi_{ik} = h^0 U_i U_k / c^2 = \mu^0 U_i U_k \tag{2.25}$$

is the kinetic energy-momentum tensor for an elastic body and:

$$\mu^0 = h^0 / c^2 \tag{2.26}$$

is the proper mass density.

Beyond the above, let us introduce in every system S the quantity:

$$\sigma_{ik} = S_{ik} - S_{i4} U_k / U_4 \tag{2.27}$$

which, on account of (2.24) and (2.25) is equal to:

$$\sigma_{ik} = T_{ik} - T_{i4} U_k / U_4 \tag{2.28}$$

From (2.1) and (2.2) the three-tensor:

$$S_{ik}^0 = \sigma_{ik}^0 = \sigma_{ik}$$

in the stationary system is a real symmetrical matrix. Also, the corresponding normalized eigenvectors $\mathbf{h}^{0(j)}$ satisfy the orthonormality relations:

$$\mathbf{h}^{(j)0} \cdot \mathbf{h}^{(\rho)0} = \delta^{je} \tag{2.29a}$$

and:

$$h_i^{(j)0} h_k^{(j)0} = \delta_{ik} \quad (j, \rho = 1,2,3) \tag{2.29b}$$

The eigenvalues $p_{(j)}^0$, the principal stresses, are the three roots of the following algebraic equation, in which λ is the unknown:

$$\left|S_{ik}^{0} - \lambda\delta_{ik}\right| = \left|\sigma_{ik}^{0} - \lambda\delta_{ik}\right| = 0 \tag{2.30}$$

The matrix S_{ik}^{0} can also be written in terms of the eigenvalues and eigenvectors as:

$$S_{ik}^{0} = \sigma_{ik}^{0} = p_{(j)}^{0} h_i^{(j)0} h_k^{(j)0} \tag{2.31}$$

From eqs. (2.23) and (2.31) one obtains the following form of the stress four-tensor in S^{o}:

$$S_{ik}^{0} = p_{(j)}^{0} h_i^{(j)0} h_k^{(j)0} \tag{2.32}$$

Thus, in any system S one has:

$$S_{ik} = p_{(j)}^{0} h_i^{(j)} h_k^{(j)} \tag{2.33}$$

From (2.24), (2.25), (2.27) and (2.33) we obtain the following expressions:

$$T_{ik} = \mu^{0} U_i U_k + p_{(j)}^{0} h_i^{(j)} h_k^{(j)} \tag{2.34}$$

$$\sigma_{ik} = S_{ik} - S_{i4} U_k / U_4 = p_{(j)}^{0} h_k^{(j)} \left(h_k^{(j)} + i h_4^{(j)} u_k / c \right) \tag{2.35}$$

By putting further:

$$h_i^{(j)} = (\mathbf{h}^{(j)}, h_4^{(j)}) \tag{2.36}$$

and introducing the notation $\mathbf{a} \bullet \mathbf{b}$ for the direct product of the vectors $\mathbf{a}$ and $\mathbf{b}$, then eqn (2.35) can be written for the relative stress tensor $\boldsymbol{\sigma}$ as following:

$$\boldsymbol{\sigma} = p_{(j)}^{0} \left[\mathbf{h}^{(j)} \bullet \mathbf{h}^{(j)} + \frac{i}{c} h_4^{(j)} (\mathbf{h}^{(j)} \bullet \mathbf{u}) \right], \ j = 1,2,3 \tag{2.37}$$

Furthermore, the triad vectors $h_i^{(j)}$ satisfy the tensor relations:

$$h_i^{(j)} h_i^{(\rho)} = \delta^{j\rho} \tag{2.38}$$

$$h_i^{(j)} h_k^{(j)} = \Delta_{ik} \tag{2.39}$$

where Δ_{ik} is given by (2.18).

If the stationary system S^{0} for every event point is chosen in such a way that the spatial axes in S^{0} and in S have the same orientation, we obtain:

$$\begin{aligned} \mathbf{h}^{(j)} &= \mathbf{h}^{(j)0} + \left\{ \mathbf{u} (\mathbf{u} \cdot \mathbf{h}^{(j)0})(\gamma - 1) \right\} / u^2 \\ h_4^{(j)} &= i \mathbf{u} \cdot \mathbf{h}^{(j)0} \, \gamma / c \end{aligned} \tag{2.40}$$

with:

$$\gamma = 1 / (1 - u^2 / c^2)^{1/2} \tag{2.41}$$

From (2.34) and (2.40) with $i = k = 4$ we have:

$$h = -T_{44} = -\mu^0 U_4^2 - p_{(j)}^0 (\mathbf{u} \cdot \mathbf{h}^{(j)0})^2 \cdot \gamma^2 / c^2 \tag{2.42}$$

In the stationary system, (2.37) reduces further to:

$$\boldsymbol{\sigma}^0 = p_{(j)}^0 \left(\mathbf{h}^{(j)0} \bullet \mathbf{h}^{(j)0} \right) \tag{2.43}$$

So, from (2.42) one obtains the following transformation law for the energy density:

$$h = \frac{h^0 + \mathbf{u} \cdot \boldsymbol{\sigma}^0 \cdot \mathbf{u} / c^2}{1 - u^2 / c^2}$$

(2.44)

$$\mathbf{u} \cdot \boldsymbol{\sigma}^0 \cdot \mathbf{u} = u_i \sigma_{ik}^0 u_k$$

and the mass density:

$$\mu = \frac{\mu^0 + \mathbf{u} \cdot \boldsymbol{\sigma}^0 \cdot \mathbf{u} / c^4}{1 - u^2 / c^2} \tag{2.45}$$

From (2.40) and (2.34) with $k = 4$, one obtains the momentum density **g** with the components $g_i = T_{i4} / ic$:

$$\mathbf{g} = \mathbf{u} \left[h^0 + \mathbf{u} \cdot \boldsymbol{\sigma}^0 \cdot \mathbf{u} (1 - \gamma^{-1}) / u^2 \right] \gamma^2 / c^2 + (\boldsymbol{\sigma}^0 \cdot \mathbf{u}) \gamma / c^2$$

(2.46)

$$(\boldsymbol{\sigma}^0 \cdot \mathbf{u})_i = \sigma_{ik}^0 u_k$$

Beyond the above, from (2.40) and (2.35) one has the relative stress tensor:

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}^0 + \mathbf{u} \bullet (\boldsymbol{\sigma}^0 \cdot \mathbf{u})(\gamma - 1) / u^2 - (\boldsymbol{\sigma}^0 \cdot \mathbf{u}) \bullet \mathbf{u} (\gamma - 1) / \gamma u^2$$

(2.47)

$$- (\mathbf{u} \bullet \mathbf{u})(\mathbf{u} \cdot \boldsymbol{\sigma}^0 \cdot \mathbf{u}) (\gamma - 1)^2 / \gamma u^4$$

In the special case **u** = (u,0,0), where the notation of the matter at the point considered is parallel to the x_1-axis (see Figs.1 and 2), the transformation equations (2.44), (2.46) and (2.47) reduce to:

$$h = \left(h^0 + \frac{u^2}{c^2} \sigma_{11}^0 \right) \gamma^2$$

$$g_{x_1} = \gamma^2 \left(\mu^0 + \frac{\sigma_{11}^0}{c^2} \right) u$$

(2.48)

$$g_{x_2} = \frac{\gamma \sigma_{21}^0}{c^2} u$$

$$g_{x_3} = \frac{\gamma \sigma_{31}^0}{c^2} u$$

and finally the relative stress tensor gives the ***Universal Equation of Elasticity***:

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} = \begin{bmatrix} \sigma_{11}^0 & \gamma\sigma_{12}^0 & \gamma\sigma_{13}^0 \\ \dfrac{1}{\gamma}\sigma_{21}^0 & \sigma_{22}^0 & \sigma_{23}^0 \\ \dfrac{1}{\gamma}\sigma_{31}^0 & \sigma_{32}^0 & \sigma_{33}^0 \end{bmatrix} \tag{2.49}$$

in which γ is given by (2.41). So, as it could be easily seen the relative stress tensor is not symmetrical, in contrast to the absolute stress tensor which is symmetrical.

3. Elastic Stress Analysis for Airframes

Let us consider the stationary frame of Fig. 1 with Γ_1 the portion of the boundary of the body on which displacements are presented, Γ_2 the surface of the body on which the force tractions are employed and Γ the total surface of the body equal to $\Gamma_1+\Gamma_2$.

Beyond the above, for the principal of virtual displacements, for linear elastic problems then the following formula is valid:

$$\int_{\Omega} (\sigma_{jk,j}^0 + b_k) u_k \,\mathrm{d}\Omega = \int_{\Gamma_2} (p_k - \overline{p}_k) u_k \,\mathrm{d}\Gamma \tag{3.1}$$

where u_k are the virtual displacements, satisfying the homogeneous boundary conditions $\overline{u}_k \equiv 0$ on Γ_1, b_k the body forces (Fig. 1) and p_k the surface tractions at the point k of the body. (Fig. 3)

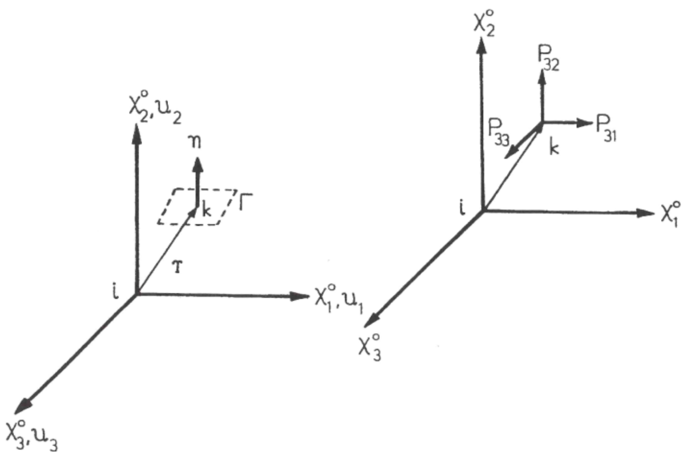

Fig. 3 The stationary system S^0.

Furthermore, (3.1) can be written as following if u_k do not satisfy the previous conditions on Γ_1:

$$\int_{\Omega} (\sigma_{jk,j}^0 + b_k) u_k \,\mathrm{d}\Omega = \int_{\Gamma_2} (p_k - \overline{p}_k) u_k \,\mathrm{d}\Gamma + \int_{\Gamma_1} (\overline{u}_k - u_k) p_k \,\mathrm{d}\Gamma \tag{3.2}$$

where $p_k = n_j \sigma_{jk}^0$ are the surface tractions corresponding to the u_k system.

By integrating (3.2) then one obtains:

$$\int_{\Omega} b_k u_k \,\mathrm{d}\Omega - \int_{\Omega} \sigma^0_{jk} \varepsilon_{jk} \,\mathrm{d}\Omega = -\int_{\Gamma_2} \bar{p}_k u_k \,\mathrm{d}\Gamma - \int_{\Gamma_1} p_k u_k \,\mathrm{d}\Gamma + \int_{\Gamma_1} (\bar{u}_k - u_k) p_k \,\mathrm{d}\Gamma \tag{3.3}$$

where ε_{jk} denote the strains.

Also, by a second integration (3.3) reduces to:

$$\begin{gathered} \int_{\Omega} b_k u_k \,\mathrm{d}\Omega + \int_{\Omega} \sigma^0_{jk,j} u_k \,\mathrm{d}\Omega = \\ -\int_{\Gamma_2} \bar{p}_k u_k \,\mathrm{d}\Gamma - \int_{\Gamma_1} p_k u_k \,\mathrm{d}\Gamma + \int_{\Gamma_1} \bar{u}_k p_k \,\mathrm{d}\Gamma + \int_{\Gamma_2} u_k p_k \,\mathrm{d}\Gamma \end{gathered} \tag{3.4}$$

On the contrary, a fundamental solution should be found, satisfying the equilibrium equations, of the following type:

$$\sigma^0_{jk,j} + \Delta^i_l = 0 \tag{3.5}$$

where Δ^i_l is the Dirac delta function which represents a unit load at i in the l direction.

The fundamental solution for a three-dimensional isotropic body can be written as: [31]

$$\begin{aligned} u^*_{lk} &= \frac{1}{16\pi G(1-v)r}\left[(3-4v)\Delta_{lk} + \frac{\vartheta r}{\vartheta x_l}\frac{\vartheta r}{\vartheta x_k}\right] \\ p^*_{lk} &= -\frac{1}{8\pi(1-v)r^2}\left[\frac{\vartheta r}{\vartheta n}\left[(1-2v)\Delta_{lk} + 3\frac{\vartheta r}{\vartheta x_l}\frac{\vartheta r}{\vartheta x_k}\right] - \right. \\ &\qquad \left. -(1-2v)\left[\frac{\vartheta r}{\vartheta x_l} n_k - \frac{\vartheta r}{\vartheta x_k} n_l\right]\right] \end{aligned} \tag{3.6}$$

in which G denotes the shear modulus, v Poisson's ratio, n the normal to the surface of the body, Δ_{lk} Kronecker's delta, r the distance from the point of application of the load to the point under consideration and n_j the direction cosines (Fig.3).

The displacements at a point are given by the formula:

$$u^i = \int_{\Gamma} up \,\mathrm{d}\Gamma - \int_{\Gamma} pu \,\mathrm{d}\Gamma + \int_{\Omega} bu \,\mathrm{d}\Omega \tag{3.7}$$

Hence, (3.7) takes the following form for the "l" component:

$$u^i_l = \int_{\Gamma} u_{lk} p_k \,\mathrm{d}\Gamma - \int_{\Gamma} p_{lk} u_k \,\mathrm{d}\Gamma + \int_{\Omega} b_k u_{lk} \,\mathrm{d}\Omega \tag{3.8}$$

By differentiating u at the internal points, we obtain the stress-tensor for an isotropic medium:

$$\sigma^0_{ij} = \frac{2Gv}{1-2v}\Delta_{ij}\frac{\vartheta u_l}{\vartheta x_l} + G\left(\frac{\vartheta u_i}{\vartheta x_j} + \frac{\vartheta u_j}{\vartheta x_i}\right) \tag{3.9}$$

Furthermore, after carrying out the differentiation we have:

$$\begin{aligned}\sigma_{ij}^{0} &= \int_{\Gamma}\left[\frac{2Gv}{1-2v}\Delta_{ij}\frac{\vartheta u_{lk}}{\vartheta x_l}+G\left(\frac{\vartheta u_{ik}}{\vartheta x_j}+\frac{\vartheta u_{jk}}{\vartheta x_i}\right)\right]p_k\,\mathrm{d}\Gamma + \\ &+\int_{\Omega}\left[\frac{2Gv}{1-2v}\Delta_{ij}\frac{\vartheta u_{lk}}{\vartheta x_l}+G\left(\frac{\vartheta u_{ik}}{\vartheta x_j}+\frac{\vartheta u_{jk}}{\vartheta x_i}\right)\right]b_k\,\mathrm{d}\Omega - \\ &-\int_{\Gamma}\left[\frac{2Gv}{1-2v}\Delta_{ij}\frac{\vartheta p_{lk}}{\vartheta x_l}+G\left(\frac{\vartheta p_{ik}}{\vartheta x_j}+\frac{\vartheta p_{jk}}{\vartheta x_i}\right)\right]u_k\,\mathrm{d}\Gamma\end{aligned} \tag{3.10}$$

Eq. (3.10) can be further written as follows:

$$\sigma_{ij}^{0} = \int_{\Gamma} D_{kij}p_k\,\mathrm{d}\Gamma - \int_{\Gamma} S_{kij}u_k\,\mathrm{d}\Gamma + \int_{\Omega} D_{kij}b_k\,\mathrm{d}\Omega \tag{3.11}$$

in which the third order tensor components D_{kij} and S_{kij} are as following:

$$D_{kij} = \frac{1}{8\pi(1-v)r^2}\left[(1-2v)\left[\Delta_{ki}r_{,j}+\Delta_{kj}r_{,i}-\Delta_{ij}r_{,k}\right]+3r_{,i}r_{,j}r_{,k}\right] \tag{3.12}$$

$$\begin{aligned}S_{kij} &= \frac{G}{4\pi(1-v)r^3}\left[3\frac{\vartheta r}{\vartheta n}\left[(1-2v)\Delta_{ij}r_{,k}+v(\Delta_{ik}r_{,j}+\Delta_{jk}r_{,i})-5r_{,i}r_{,j}r_{,k}\right]\right. \\ &\left.+3v(n_i r_{,j}r_{,k}+n_j r_{,i}r_{,k})+(1-2v)(3n_k r_{,i}r_{,j}+n_j\Delta_{ik}+n_i\Delta_{jk})-(1-4v)n_k\Delta_{ij}\right]\end{aligned} \tag{3.13}$$

with: $r_{,i} = \dfrac{\vartheta r}{\vartheta x_i}$

Finally, because of eqs (2.49) and (3.11) by considering the moving system S of Fig. 2, then the stress-tensor reduces to the form:

$$\begin{aligned}\sigma_{11} &= \sigma_{11}^{0} \\ \sigma_{12} &= \gamma\sigma_{12}^{0} \\ \sigma_{13} &= \gamma\sigma_{13}^{0} \\ \sigma_{21} &= \frac{1}{\gamma}\sigma_{21}^{0} \\ \sigma_{22} &= \sigma_{22}^{0} \\ \sigma_{23} &= \sigma_{23}^{0} \\ \sigma_{31} &= \frac{1}{\gamma}\sigma_{31}^{0} \\ \sigma_{32} &= \sigma_{32}^{0} \\ \sigma_{33} &= \sigma_{33}^{0}\end{aligned} \tag{3.14}$$

where σ_{ij}^{0} are given by. (3.11) to (3.13).

The following Table 1 shows the values of γ as given by (2.41) for some arbitrary values of the velocity u for the next generation aircraft or spacecraft:

Table 1

Velocity u	$\gamma = 1/\sqrt{1-u^2/c^2}$	Velocity u	$\gamma = 1/\sqrt{1-u^2/c^2}$
50,000 *km/h*	1.000000001	0.800c	1.666666667
100,000 *km/h*	1.000000004	0.900c	2.294157339
200,000 *km/h*	1.000000017	0.950c	3.202563076
500,000 *km/h*	1.000000107	0.990c	7.088812050
10E+06 *km/h*	1.000000429	0.999c	22.36627204
10E+07 *km/h*	1.000042870	0.9999c	70.71244596
10E+08 *km/h*	1.004314456	0.99999c	223.6073568
2x10E+8 *km/h*	1.017600788	0.999999c	707.1067812
c/3	1.060660172	0.9999999c	2236.067978
c/2	1.154700538	0.99999999c	7071.067812
2c/3	1.341640786	0.999999999c	22360.67978
3c/4	1.511857892	c	∞

Thus, from the above Table follows that for small velocities 50,000 *km*/*h* to 200,000 *km*/*h*, the absolute and the relative stress tensor are nearly the same. On the other hand, for bigger velocities like $c/3$, $c/2$, $3c/4$, or 0.80c (c = speed of light), the variable γ takes values more than the unit and thus, relative stress tensor is very different from the absolute one. Finally, for values of the velocity of the moving structure near the speed of light, the variable γ takes bigger values, while when the velocity is equal to the speed of light, then γ tends to the infinity.

Hence, the Singular Integral Operators Method (S.I.O.M.) as was proposed by E.G.Ladopoulos [4], [8], [9], [11], [12], [13], [15] and E.G.Ladopoulos et al. [22] will be used for the numerical evaluation of the stress tensor (3.11), for every specific case.

4. Fracture Mechanics Analysis for Next Generation Aircraft & Spacecraft

The first and second mode stress intensity factors in the stationary frame for elastic materials in an in-plane loaded plate are given by the relations (Fig.4): [61]

$$K_I^0 = \lim_{x_1 \to 0} \left\{ \sqrt{2\pi x_1}\, \sigma_{22}^0 \right\} \tag{4.1}$$

$$K_{II}^0 = \lim_{x_1 \to 0} \left\{ \sqrt{2\pi x_1}\, \sigma_{12}^0 \right\} \tag{4.2}$$

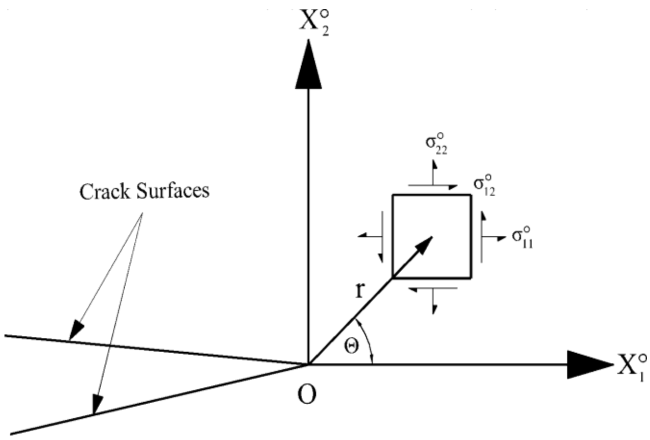

Fig. 4 2-D Coordinates near the crack tip.

Furthermore, the first and second mode stress intensity factors for the airframes are equal to:

$$K_I = \lim_{x_1 \to 0} \left\{ \sqrt{2\pi x_1}\, \sigma_{22} \right\} \tag{4.3}$$

$$K_{II} = \lim_{x_1 \to 0} \left\{ \sqrt{2\pi x_1}\, \sigma_{12} \right\} \tag{4.4}$$

Thus, because of (3.14), eqs (4.3) and (4.4) can be written as:

$$K_I = \lim_{x_1 \to 0} \left\{ \sqrt{2\pi x_1}\, \sigma_{22}^0 \right\} \tag{4.5}$$

$$K_{II} = \lim_{x_1 \to 0} \left\{ \sqrt{2\pi x_1}\, \gamma \sigma_{12}^0 \right\} \tag{4.6}$$

On the other hand, the first, second and third mode stress intensity factors in the stationary frame for elastic materials in a 3-D solid are given by the relations (Fig.5): [62]

$$K_I^0 = \lim_{x_1 \to 0} \left\{ \sqrt{2\pi x_1}\, \sigma_{22}^0 \right\} \tag{4.7}$$

$$K_{II}^0 = \lim_{x_1 \to 0} \left\{ \sqrt{2\pi x_1}\, \sigma_{12}^0 \right\} \tag{4.8}$$

$$K_{III}^0 = \lim_{x_1 \to 0} \left\{ \sqrt{2\pi x_1}\, \sigma_{23}^0 \right\} \tag{4.9}$$

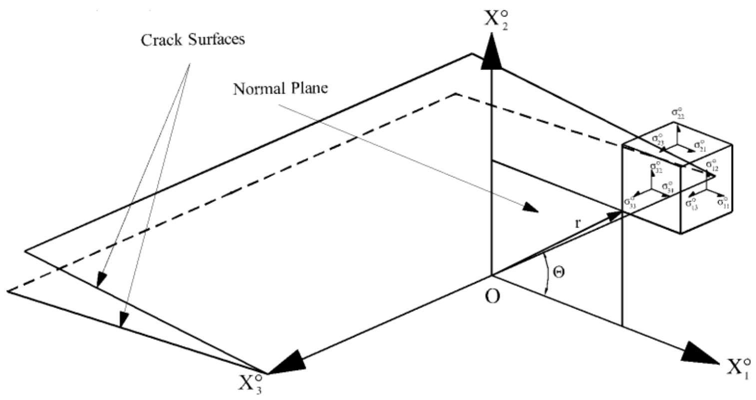

Fig. 5 3-D Coordinates near the crack tip.

Beyond the above, the first, second and third mode stress intensity factors, referred as the ***"Universal Stress Intensity Factors"*** for the airframes, are equal to:

$$K_I = \lim_{x_1 \to 0} \left\{ \sqrt{2\pi x_1}\, \sigma_{22} \right\} \tag{4.10}$$

$$K_{II} = \lim_{x_1 \to 0} \left\{ \sqrt{2\pi x_1}\, \sigma_{12} \right\} \tag{4.11}$$

$$K_{III} = \lim_{x_1 \to 0} \left\{ \sqrt{2\pi x_1}\, \sigma_{23} \right\} \tag{4.12}$$

Hence, because of (3.14), eqs (4.10), (4.11) and (4.12) can be written as:

$$K_I = \lim_{x_1 \to 0} \left\{ \sqrt{2\pi x_1}\, \sigma_{22}^0 \right\} \tag{4.13}$$

$$K_{II} = \lim_{x_1 \to 0} \left\{ \sqrt{2\pi x_1} \gamma \sigma_{12}^0 \right\} \quad (4.14)$$

$$K_{III} = \lim_{x_1 \to 0} \left\{ \sqrt{2\pi x_1} \sigma_{23}^0 \right\} \quad (4.15)$$

Finally, from eqs (4.13) to (4.15) follows that the first and third mode stress intensity factors are the same for both stationary and moving frames, while the second mode stress intensity factor is much different in the above frames. So, because of the difference of the second mode stress intensity factor, follows that the fracture behavior of the next generation aircraft and spacecraft, which is calculated by using the *"Universal Stress Intensity Factors"*, will be much different and thus special materials should be used for their construction.

5. Conclusions

By the present research in the area of aeronautics technologies the theory of *“Relativistic Elasticity”* was investigated and applied for the design of a new generation large aircraft with speeds in the range of 50,000 km/h. Such a design and construction of a new generation aircraft will be applied to an increased market share of International Aeronautical Industries all over the world. Moreover, the theory of *“Relativistic Elasticity”* was applied for the design of the next generation spacecraft moving with very high speeds, even approaching the speed of light, as the target of the International Space Agencies is to achieve such spacecraft in the future, which should be as closer as possible.

The future investigation concerns to the determination of the proper composite materials or any other kind of materials for the construction of the next generation spacecfract, as usual composite solids are not proper for such a construction. Also, the need for lighter, more affordable high performance aircraft and spacecraft has accelerated demand for new advanced concepts. For example composite solids like Fibre Metal Laminates would be ideal to icrease the fatigue characteristics of the laminated metal structures by adding fibres in the bond line.

The theory of *“Relativistic Elasticity”* and correspondingly the *“Universal Equation of Elasticity”* show that there is a considerable difference between the absolute stress tensor of the airframe even in the range of speeds of 50,000 km/h. For bigger speeds the difference between the two stress tensors is very much increased. *“Relativistic Mechanics”* results as a combination of the theories of "Classical Elasticity" and "Special Relativity".

Also, for the structural design of the next generation aircraft and spacecraft will be used the stress tensor of the airframe in combination to the singular integral equations. Such a stress tensor is reduced to the solution of a multidimensional singular integral equation and for its numerical evaluation will be used the Singular Integral Operators Method (S.I.O.M.). Finally, the fracture mechanics analysis of the next generation aircraft and spacecraft has shown that the second mode stress intensity factor is much different than in the stationary frame and consequently the *"Universal Stress Intensity Factors"* are valid.

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