Universal Journal of Aeronautical & Aerospace Sciences **1 (2013), 1-17**
www.papersciences.com

Next Generation Aircraft & Spacecraft by Relativistic Elasticity & Thermo-Elasticity

E.G. Ladopoulos
Interpaper Research Organization
8, Dimaki Str.
Athens, GR - 106 72, Greece
eladopoulos@interpaper.org

Abstract
The theories of *"Relativistic Elasticity"* and *"Relativistic Thermo-Elasticity"* is proposed for the design of the new generation large aircraft with turbojet engines and speeds in the range of 50,000 km/h and for the new generation spacecraft of any speed. This theory shows that there is a considerable difference between the absolute stress tensor and the stress tensor of the moving frame even in the range of speeds of 50,000 km/h. For bigger speeds of a next generation spacecraft, like c/3, c/2 or 3c/4 (c=speed of light), the difference between the two stress tensors is very much increased. Therefore, for the next generation spacecraft with very high speeds, then the relative stress tensor will be very much different than the absolute stress tensor. Beyond the above, for velocities near the speed of light, then the values of the relative stress tensor are very much bigger than the corresponding values of the absolute stress tensor. The proposed theory of *"Relativistic Elasticity"* is a combination between the theories of "Classical Elasticity" and "Special Relativity" and results to the ***"Universal Equation of Elasticity"***. Also, the proposed theory of *"Relativistic Thermo-Elasticity"* is a combination between the theories of "Classical Thermo-Elasticity" and "Special Relativity" and results to the ***"Universal Equation of Thermo-Elasticity".*** For the structural design of the new generation aircraft and spacecraft the stress tensor of the airframe will be used in combination to the singular integral equations method. Such a stress tensor is reduced to the solution of a multidimensional singular integral equation and for its numerical evaluation will be used the *Singular Integral Operators Method (S.I.O.M.).*

Key Word and Phrases
Relativistic Elasticity, Relativistic Thermo-Elasticity, Aircraft, Spacecraft, Relative Stress Tensor, Absolute Stress Tensor, Stationary and Moving Frames, Energy-Momentum Tensor, Multidimensional Singular Integral Equations, Singular Integral Operators Method (S.I.O.M.), Universal Equation of Elasticity, Universal Equation of Thermo-Elasticity.

1. Aircraft and Spacecraft of the Next Generation

The current target of international Aeronautical Industries is to achieve a competitive technological advantage in certain strategic areas of new and rapidly developing advanced technologies, by which in the longer terms, can be achieved increased market share. This considerably big market share includes the design of a new generation large aircraft with speeds even in the range of 50,000 km/h. The application of new generation turbojet engines makes possible the design of such type of large aircraft and therefore there is a need of elastic stress analysis for the construction of the total parts of such type of new generation aircraft.

Furthermore, the target of the International Space Agencies is to achieve in the future, next generation spacecraft moving with very high speeds, even approaching the speed of light. How far is this future ? According to the present research this future could be much closer than everybody believes. In these cases of the next generation spacecraft the relative stress tensor will be much different than the absolute stress tensor and so special material should be used for the construction of such spacecraft. The type of the proper solid for the construction of the next generation spacecraft is under investigation and will be very much different than the usual composite materials.

In the present investigation we will show that there is a significant difference between the absolute stress tensor and the stress tensor of the airframe even in the range of speeds of 50,000 km/h.

On the other hand, for bigger speeds the difference of the two stress tensors is very much increased. Thus, for bigger velocities like c/3, c/2 or 3c/4 (c=speed of light) the relative stress tensor is very much different than the absolute one, while for velocities near the speed of light the values of the relative stress tensor are much bigger than the corresponding values of the absolute stress tensor. The study of the connection between the stress tensors of the absolute frame and the airframe is included in the theory proposed by E.G.Ladopoulos [30] - [32] under the term *"Relativistic Elasticity"* and *"Relativistic Thermo-Elasticity"* and the final formula which results from the above theories is called the ***"Universal Equation of Elasticity"*** and the***"Universal Equation of Thermo-Elasticity"***, correspondingly. Hence, in the present investigation the theories of *"Relativistic Elasticity"* and *"Relativistic Thermo-Elasticity"* will be applied for the elastic stress analysis design of the next generation aircraft and spacecraft.

Furthermore, E.G.Ladopoulos [1]-[16] and E.G.Ladopoulos et al. [17]-[22] proposed several linear singular integral equation methods applied to elasticity, plasticity and fracture mechanics applications. In the above studies the Singular Integral Operators Method (S.I.O.M.) is investigated for the numerical evaluation of the multidimensional singular integral equations in which is reduced the stress tensor analysis of the linear elastic theory. Also, the theory of linear singular integral equations was extended to non-linear singular integral equations, too. [23]-[29]. The theories of *"Relativistic Elasticity"* and *"Relativistic Thermo-Elasticity"* will be applied to the design of the elastic stress analysis for the airframes. *"Relativistic Elasticity"* and *"Relativistic Thermo-Elasticity"* are derived as a generalization of the classical theories of elastic and thermo-elastic stress analysis for stationary frames, correspondingly. For future aerospace applications the difference between the relative and the absolute stress tensors will be of increasing interest. Also, the classical theory of elastic stress analysis and thermo-elastic stress analysis began to be analyzed in the early nineteenth century and was further developed in the twentieth century. In the past were written several important monographs on the classical theory of elasticity and thermo-elasticity. [33]-[52].

On the other hand, over the past years special attention has been concentrated on the theoretical aspects of the special theory of relativity. Hence, some classical monographs were written, dealing with the theoretical foundations and investigations of the special and the general theory of relativity. [53]–[60]. Beyond the above, a very important point which will be shown in the present investigation is that the **"*relative stress tensor is not symmetrical*"**, while, as it is well known, the **"*absolute stress tensor is symmetrical*"**. This difference is very important for the design of the next generation aircraft and spacecraft of very high speeds. Thus, the foundations of the theory of *"Relativistic Elasticity"* for airstructures lead to a general theory, in which no restriction is made with regard to the relative motion. This general theory is further reduced to one class of relative motion, uniform in direction and velocity.

2. Relativistic Elastic Stress Analysis

The state of stress at a point in the stationary frame S^0, is defined by the following symmetrical stress tensor: (Fig.1)

$$\sigma^0 = \begin{bmatrix} \sigma_{11}^0 & \sigma_{12}^0 & \sigma_{13}^0 \\ \sigma_{21}^0 & \sigma_{22}^0 & \sigma_{23}^0 \\ \sigma_{31}^0 & \sigma_{32}^0 & \sigma_{33}^0 \end{bmatrix} \tag{2.1}$$

in which: $$\sigma_{21}^0 = \sigma_{12}^0, \sigma_{31}^0 = \sigma_{13}^0, \sigma_{32}^0 = \sigma_{23}^0 \tag{2.2}$$

Let us consider an infinitesimal face element df with a directed normal, defined by a unit vector $\mathbf{n}$, at definite point p in the three-space of a Lorenz system. The matter on either side of this face element experiences a force which is proportional to df.

Hence, the force is valid as:

$$\mathrm{d}\boldsymbol{\sigma}(\mathbf{n}) = \boldsymbol{\sigma}(\mathbf{n})\,\mathrm{d}f \tag{2.3}$$

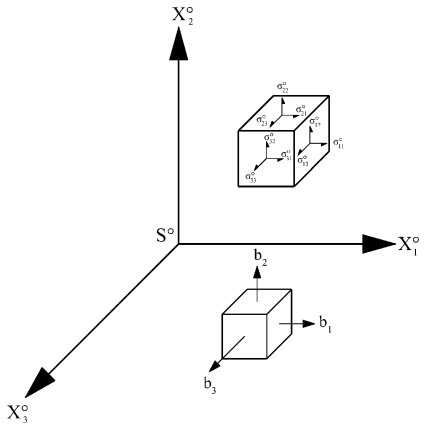

Fig. 1 The state of stress σ_{ik}^{0} in the stationary system S^{0}.

The components $\sigma_i(\mathbf{n})$ of $\boldsymbol{\sigma}(\mathbf{n})$ are linear functions of the components n_k of $\mathbf{n}$:

$$\sigma_i(\mathbf{n}) = \sigma_{ik} n_k\,,\ i,k = 1,2,3 \tag{2.4}$$

where σ_{ik} is the elastic stress tensor, which can be also called the relative stress tensor, in contrast to the space part σ_{ik}^{0} of the total energy-momentum tensor T_{ik}, referred as the absolute stress tensor. [53], [54} (Fig. 2).

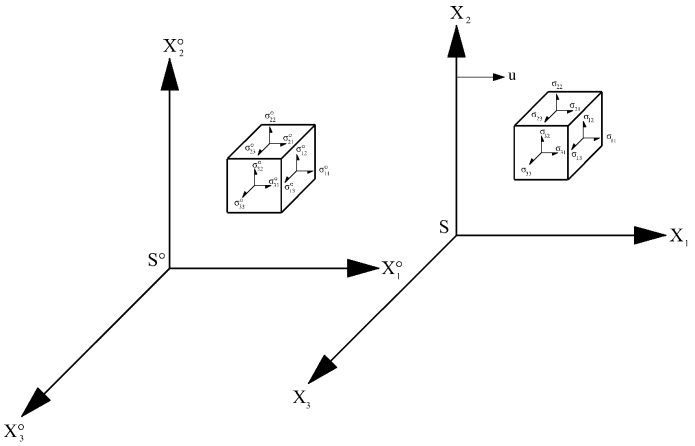

Fig. 2 The state of stress σ_{ik}^{0} in the stationary system S^{0} and σ_{ik} in the airframe system with velocity u parallel to the x_1 - axis.

Beyond the above, the connection between the absolute and relative stress tensors is defined as:

$$\sigma_{ik}^{0} = \sigma_{ik} + g_i u_k\,,\ i,k = 1,2,3 \tag{2.5}$$

where g_i are the components of the momentum density $\mathbf{g}$ and u_k the components of the velocity $\mathbf{u}$ of the matter.

Furthermore, the connection between $\mathbf{g}$ and the energy flux $\mathbf{s}$, is valid as:

$$\mathbf{g} = \mathbf{s}/c^2 \tag{2.6}$$

in which c denotes the speed of light (= 300.000 *km*/*sec*).

The total work done per unit time by elastic forces on the matter inside the closed surface f is given by the formula:

$$W = \int_f \big(\boldsymbol{\sigma}(\mathbf{n})\cdot\mathbf{u}\big)\mathrm{d}f = \int_f \sigma_{ik} n_k u_i \,\mathrm{d}f = -\int_\upsilon \frac{\vartheta(u_i\sigma_{ik})}{\vartheta x_k}\mathrm{d}\upsilon\ ,\ i,k = 1,2,3 \tag{2.7}$$

where the integration in the last integral is extended over the interior υ of the surface f.

Thus, the work done on an infinitesimal piece of matter of volume $\delta\upsilon$ is valid as:

$$\delta W = -\frac{\vartheta(u_i\sigma_{ik})}{\vartheta x_k}\delta\upsilon \tag{2.8}$$

Also, (2.8) must be equal to the increase per unit time of the energy inside $\delta\upsilon$:

$$\frac{\mathrm{d}}{\mathrm{d}t}(h\delta\upsilon) = \delta W \tag{2.9}$$

where h is the total energy density, including the elastic energy and $\mathrm{d}/\mathrm{d}t$ denotes the substantial time derivative.

Eq. (2.9) is valid as:

$$\frac{\mathrm{d}}{\mathrm{d}t}(h\delta\upsilon) = \left(\frac{\vartheta h}{\vartheta t} + \frac{\vartheta h}{\vartheta x_k}u_k\right)\delta\upsilon + h\delta\upsilon\frac{\vartheta u_k}{\vartheta x_k} = \left[\frac{\vartheta h}{\vartheta t} + \frac{\vartheta}{\vartheta x_k}(hu_k)\right]\delta\upsilon \tag{2.10}$$

which finally leads to the relation:

$$\frac{\vartheta h}{\vartheta t} + \frac{\vartheta}{\vartheta x_k}(hu_k + u_i\sigma_{ik}) = 0 \tag{2.11}$$

So, the total energy flow is valid as:

$$\mathbf{s} = \mathbf{h}\mathbf{u} + (\mathbf{u}\cdot\boldsymbol{\sigma}) \tag{2.12}$$

where $(\mathbf{u}\cdot\boldsymbol{\sigma})$ is a space vector with components $(\mathbf{u}\cdot\boldsymbol{\sigma})_k = u_i\sigma_{ik}$.

Thus, the total momentum density can be written as:

$$\mathbf{g} = \frac{\mathbf{s}}{c^2} = \mu\mathbf{u} + \frac{(\mathbf{u}\cdot\boldsymbol{\sigma})}{c^2} \tag{2.13}$$

where $\mu = h/c^2$ is the total mass density, including the mass of the elastic energy.

From (2.5) and (2.13) one obtains:

$$\sigma_{ik} - \sigma_{ki} = -g_i u_k + g_k u_i = [-(\mathbf{u}\cdot\boldsymbol{\sigma})_i u_k + (\mathbf{u}\cdot\boldsymbol{\sigma})_k u_i]/c^2 \neq 0 \tag{2.14}$$

which shows that the relative stress tensor is not symmetrical, in contrast to the absolute stress tensor (2.1) which is symmetrical.

In the stationary frame S^0 the velocity $u^0 = 0$ and hence, from (2.5), (2.12) and (2.13) we obtain the following expressions:

$$\sigma_{ik}^0 = \sigma_{ik} = \sigma_{ki} = \sigma_{ki}^0 \ (i,k = 1,2,3) \tag{2.15}$$

Beyond the above, the mechanical energy-momentum tensor satisfies the following relation:

$$T_{ik}U_k = -h^0 U_i \tag{2.16}$$

where U_i is the four-velocity of the matter, in the Lorentz system and $U_i^0 = (0,0,0,ic)$.

So, the following scalar can be formed:

$$U_i T_{ik} U_k / c^2 = U_i^0 T_{ik}^0 U_k^0 / c^2 = -T_{44}^0 = h^0(x_1) \tag{2.17}$$

with $h^0(x_1)$ the invariant rest energy density considered as a scalar function of the coordinates (x_i) $(i = 1,2,3)$ in S. (Fig. 2)

By applying further the tensor:

$$\Delta_{ik} = \delta_{ik} + U_i U_k / c^2 \tag{2.18}$$

which satisfies the relations:

$$U_i \Delta_{ik} = \Delta_{ik} U_k = 0 \tag{2.19}$$

then, we can form the following symmetrical tensor:

$$S_{ik} = \Delta_{i1} T_{1m} \Delta_{mk} = S_{ki} \tag{2.20}$$

which is orthogonal to U_i:

$$U_i S_{ik} = S_{ik} U_k = 0 \tag{2.21}$$

By combining eqs. (2.16), (2.17) and (2.20) we have:

$$S_{ik} = T_{ik} - h^0 U_i U_k / c^2 \tag{2.22}$$

Furthermore, in the stationary system S_0 one has:

$$S_{ik}^0 = \sigma_{ik}^0 = \sigma_{ik}, \ S_{i4}^0 = S_{4i}^0 = 0 \tag{2.23}$$

Eq. (2.22) may also be written as:

$$T_{ik} = \xi_{ik} + S_{ik} \tag{2.24}$$

where:

$$\xi_{ik} = h^0 U_i U_k / c^2 = \mu^0 U_i U_k \tag{2.25}$$

is the kinetic energy-momentum tensor for an elastic body and:

$$\mu^0 = h^0 / c^2 \tag{2.26}$$

is the proper mass density.

Furthermore, we introduce in every system S the quantity:

$$\sigma_{ik} = S_{ik} - S_{i4} U_k / U_4 \tag{2.27}$$

which, on account of (2.24) and (2.25) is valid as:

$$\sigma_{ik} = T_{ik} - T_{i4} U_k / U_4 \tag{2.28}$$

From (2.1) and (2.2) the three-tensor:

$$S_{ik}^0 = \sigma_{ik}^0 = \sigma_{ik}$$

in the stationary system is a real symmetrical matrix. The corresponding normalized eigenvectors $\mathbf{h}^{0(j)}$ satisfy the orthonormality relations:

$$\mathbf{h}^{(j)0} \cdot \mathbf{h}^{(\rho)0} = \delta^{je} \tag{2.29a}$$

and:

$$h_i^{(j)0} h_k^{(j)0} = \delta_{ik} \quad (j, \rho = 1,2,3) \tag{2.29b}$$

The eigenvalues $p_{(j)}^0$, the principal stresses, are the three roots of the following algebraic equation, where λ is the unknown:

$$\left| S_{ik}^0 - \lambda \delta_{ik} \right| = \left| \sigma_{ik}^0 - \lambda \delta_{ik} \right| = 0 \tag{2.30}$$

The matrix S_{ik}^0 can also be written in terms of the eigenvalues and eigenvectors as:

$$S_{ik}^0 = \sigma_{ik}^0 = p_{(j)}^0 h_i^{(j)0} h_k^{(j)0} \tag{2.31}$$

From eqs. (2.23) and (2.31) we obtain the following form of the stress four-tensor in S^o:

$$S_{ik}^0 = p_{(j)}^0 h_i^{(j)0} h_k^{(j)0} \tag{2.32}$$

So, in any system S we have

$$S_{ik} = p^0_{(j)} h^{(j)}_i h^{(j)}_k \tag{2.33}$$

From (2.24), (2.25), (2.27) and (2.33) one obtains the following expressions:

$$T_{ik} = \mu^0 U_i U_k + p^0_{(j)} h^{(j)}_i h^{(j)}_k \tag{2.34}$$

$$\sigma_{ik} = S_{ik} - S_{i4} U_k / U_4 = p^0_{(j)} h^{(j)}_k \left(h^{(j)}_k + i h^{(j)}_4 u_k / c \right) \tag{2.35}$$

By putting:

$$h^{(j)}_i = (\mathbf{h}^{(j)}, h^{(j)}_4) \tag{2.36}$$

and introducing the notation $\mathbf{a} \bullet \mathbf{b}$ for the direct product of the vectors $\mathbf{a}$ and $\mathbf{b}$, then eqn (2.35) may be written for the relative stress tensor $\boldsymbol{\sigma}$ as:

$$\boldsymbol{\sigma} = p^0_{(j)} \left[\mathbf{h}^{(j)} \bullet \mathbf{h}^{(j)} + \frac{i}{c} h^{(j)}_4 (\mathbf{h}^{(j)} \bullet \mathbf{u}) \right], j = 1,2,3 \tag{2.37}$$

Also, the triad vectors $h^{(j)}_i$ satisfy the tensor relations:

$$h^{(j)}_i h^{(\rho)}_i = \delta^{j\rho} \tag{2.38}$$

$$h^{(j)}_i h^{(j)}_k = \Delta_{ik} \tag{2.39}$$

with Δ_{ik} given by (2.18).

If the stationary system S^0 for every event point is chosen in such a way that the spatial axes in S^0 and in S have the same orientation, one obtains:

$$\begin{aligned} \mathbf{h}^{(j)} &= \mathbf{h}^{(j)0} + \left\{ \mathbf{u} (\mathbf{u} \cdot \mathbf{h}^{(j)0}) (\gamma - 1) \right\} / u^2 \\ h^{(j)}_4 &= i \mathbf{u} \cdot \mathbf{h}^{(j)0} \gamma / c \end{aligned} \tag{2.40}$$

with:

$$\gamma = 1 / (1 - u^2 / c^2)^{1/2} \tag{2.41}$$

From (2.34) and (2.40) with $i = k = 4$ one has:

$$h = -T_{44} = -\mu^0 U^2_4 - p^0_{(j)} (\mathbf{u} \cdot \mathbf{h}^{(j)0})^2 \cdot \gamma^2 / c^2 \tag{2.42}$$

In the stationary system, (2.37) reduces to:

$$\boldsymbol{\sigma}^0 = p^0_{(j)} \left(\mathbf{h}^{(j)0} \bullet \mathbf{h}^{(j)0} \right) \tag{2.43}$$

Thus, from (2.42) we obtain the following transformation law for the energy density:

$$h = \frac{h^0 + \mathbf{u}\cdot\boldsymbol{\sigma}^0\cdot\mathbf{u}/c^2}{1-u^2/c^2}$$
$$\mathbf{u}\cdot\boldsymbol{\sigma}^0\cdot\mathbf{u} = u_i\sigma_{ik}^0 u_k \tag{2.44}$$

and the mass density:

$$\mu = \frac{\mu^0 + \mathbf{u}\cdot\boldsymbol{\sigma}^0\cdot\mathbf{u}/c^4}{1-u^2/c^2} \tag{2.45}$$

From (2.40) and (2.34) with k = 4, we obtain the momentum density $\mathbf{g}$ with the components $g_i = T_{i4}/ic$:

$$\mathbf{g} = \mathbf{u}\left[h^0 + \mathbf{u}\cdot\boldsymbol{\sigma}^0\cdot\mathbf{u}(1-\gamma^{-1})/u^2\right]\gamma^2/c^2 + (\boldsymbol{\sigma}^0\cdot\mathbf{u})\gamma/c^2$$
$$(\boldsymbol{\sigma}^0\cdot\mathbf{u})_i = \sigma_{ik}^0 u_k \tag{2.46}$$

Also, from (2.40) and (2.35) we have the relative stress tensor:

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}^0 + \mathbf{u}\bullet(\boldsymbol{\sigma}^0\cdot\mathbf{u})(\gamma-1)/u^2 - (\boldsymbol{\sigma}^0\cdot\mathbf{u})\bullet\mathbf{u}(\gamma-1)/\gamma u^2$$
$$-(\mathbf{u}\bullet\mathbf{u})(\mathbf{u}\cdot\boldsymbol{\sigma}^0\cdot\mathbf{u})(\gamma-1)^2/\gamma u^4 \tag{2.47}$$

In the special case $\mathbf{u}$ = (u,0,0), where the notation of the matter at the point considered is parallel to the x_1-axis (see Figs.1 and 2), the transformation equations (2.44), (2.46) and (2.47) reduce to:

$$h = \left(h^0 + \frac{u^2}{c^2}\sigma_{11}^0\right)\gamma^2$$
$$g_{x_1} = \gamma^2\left(\mu^0 + \frac{\sigma_{11}^0}{c^2}\right)u$$
$$g_{x_2} = \frac{\gamma\sigma_{21}^0}{c^2}u \tag{2.48}$$
$$g_{x_3} = \frac{\gamma\sigma_{31}^0}{c^2}u$$

and the relative stress tensor gives the ***Universal Equation of Elasticity***:

$$\boldsymbol{\sigma} = \begin{bmatrix}\sigma_{11} & \sigma_{12} & \sigma_{13}\\ \sigma_{21} & \sigma_{22} & \sigma_{23}\\ \sigma_{31} & \sigma_{32} & \sigma_{33}\end{bmatrix} = \begin{bmatrix}\sigma_{11}^0 & \gamma\sigma_{12}^0 & \gamma\sigma_{13}^0\\ \frac{1}{\gamma}\sigma_{21}^0 & \sigma_{22}^0 & \sigma_{23}^0\\ \frac{1}{\gamma}\sigma_{31}^0 & \sigma_{32}^0 & \sigma_{33}^0\end{bmatrix} \tag{2.49}$$

where γ is given by (2.41). Finally, as it could be easily seen the relative stress tensor is not symmetrical, in contrast to the absolute stress tensor which is symmetrical.

2. Relativistic Thermo-Elastic Stress Analysis

In the previous section it has been regarded our system, the elastic body, as a purely mechanical system. However, all macroscopic systems are in reality thermo-dynamical systems with properties depending on non-mechanical variables such as the proper temperature T^o, and the question which arises is to what kind of thermodynamical processes may be described by an energy-momentum tensor.

Thus, it is clear that all properties in which heat energy is transferred from one part of the system to another are excluded, for heat flow in the manner would give rise to a non-vanishing energy current in the rest system.

Let us consider a general system of continuously distributed ponderable or visible matter, inside which invisible heat conduction can take place, while the motion of the visible matter is described by the four-velocity U_i. Then the energy-momentum tensor of the general system is given by the relation:

$$T_{ik} = M_{ik} + H_{ik} \tag{3.1}$$

where M_{ik} is the mechanical part of the energy-momentum tensor and H_{ik} the heat part.

Beyond the above, the mechanical part M_{ik} is valid by the following formula:

$$M_{ik} = d^0 U_i U_k / c^2 + S_{ik} \tag{3.2}$$

and the heat part:

$$H_{ik} = \left(U_i V_k + V_i U_k\right)/c^2 \tag{3.3}$$

where the four-vector V_i satisfies the relation:

$$V_i = -\Delta_{ik} T_{kj} U_j = -T_{ik} U_k - d^0 U_i \tag{3.4}$$

in which d^0 denote the normalized eigenvectors, Δ_{ik} is the tensor given by (2.18) and P_{ik} the potential part of the energy momentum tensor.

The four-vector V_i is orthogonal to U_i:

$$U_i V_i = 0 \tag{3.5}$$

and so one obtains:

$$V_i = \left(\mathbf{V}, i\left(\mathbf{V},\mathbf{u}\right)/c\right) \tag{3.6}$$

where $\mathbf{u}$ denotes the velocity of the matter.

So, in the stationary system, (3.6) reduces to:

$$V_i^0 = \left(\mathbf{V}^0, 0\right) \tag{3.7}$$

Moreover, by replacing (2.18) into (2.20) and using (2.17) and (3.4), then we obtain instead of (2.22):

$$S_{ik} = T_{ik} - d^0 U_i U_k / c^2 - \left(U_i V_k + V_i U_k\right)/c^2 \tag{3.8}$$

Therefore, from (3.8) follows the required relation (3.1), instead of (2.24).

Furthermore, consider the general system of continuously matter described previously inside which invisible heat conduction can take place, while the motion of the matter is described by the four-velocity U_i or by the velocity u_i.

Then, the connection between the energy-momentum tensor T_{ik} and the relative stress tensor σ_{ik} of the general system is given by the relation:

$$T_{ik} = g_i u_k + \sigma_{ik} + u_i \xi_k / c^2 \tag{3.9}$$

with:

$$\xi_k = U_4 \left(V_k - V_4 U_k / U_4\right)/ic \tag{3.10}$$

where V_k denotes the four-vector given by (3.4), g_i the momentum density and c the speed of light.

The quantity ξ_k seems to be the most important part of ξ_{ik}:

$$\xi_{ik} = H_{ik} - H_{i4} U_k / U_4 = U_i \left(V_k - V_4 U_k / U_4\right)/c^2 \tag{3.11}$$

Also, ξ_k can be written by the following form by using (2.41) and (3.6):

$$\xi_k = \left(\boldsymbol{\xi}, 0\right) \tag{3.12}$$

with:

$$\boldsymbol{\xi} = \gamma\left[\mathbf{V} - \mathbf{u}\left(\mathbf{V},\mathbf{u}\right)/c^2\right] \tag{3.13}$$

Beyond the above, in the stationary system, $\boldsymbol{\xi}^0$ is equal to the heat current density $\mathbf{V}^0$:

$$\boldsymbol{\xi}^0 = \mathbf{V}^0 \tag{3.14}$$

By combining (3.10) and (3.11), then one obtains:

$$\xi_{ik} = U_i\,\xi_k/\gamma c^2 \tag{3.15}$$

Thus, by using (2.35), (3.1), (3.2), (3.11) and (3.15), we have:

$$T_{ik} - T_{i4}\,U_k/U_4 = \sigma_{ik} + \xi_{ik} = \sigma_{ik} + U_i\,\xi_k/\gamma c^2 \tag{3.16}$$

which finally reduces to the required formula (3.9).

Furthermore, we consider the general system of continuously matter, inside which invisible heat conduction can take place. Then the momentum density $\mathbf{g}$ of this system is given by the ***Universal Equation of Thermo-Elasticity:***

$$\mathbf{g} = m\mathbf{u} + \frac{\left(\mathbf{u},\boldsymbol{\sigma}\right)}{c^2} + \frac{\boldsymbol{\xi}}{c^2} \tag{3.17}$$

where $\mathbf{u}$ denotes the velocity of the matter at the place and time considered, $\boldsymbol{\sigma}$ the relative stress tensor, $\boldsymbol{\xi}$ is given by (3.13) and $m = E/c^2$ is the total mass density.

From (3.9), we obtain for the energy current density:

$$D_k = Eu_k + u_i\sigma_{ik} + \xi_k \tag{3.18}$$

which can be further written as:

$$\mathbf{D} = E\mathbf{u} + \left(\mathbf{u},\boldsymbol{\sigma}\right) + \boldsymbol{\xi} \tag{3.19}$$

Hence, from (3.19) by using the formula of the momentum density $\mathbf{g}$:

$$\mathbf{g} = \mathbf{D}/c^2 \tag{3.20}$$

we obtain the required relation (3.17) which is a generalization, for a general system with heat conduction.

4. Elastic Stress Analysis for Next Generation Aircraft & Spacecraft

Consider the stationary frame of Fig. 1 with Γ_1 the portion of the boundary of the body on which displacements are presented, Γ_2 the surface of the body on which the force tractions are employed and Γ the total surface of the body equal to $\Gamma_1+\Gamma_2$.

Furthermore, for the principal of virtual displacements, for linear elastic problems then the following formula is valid:

$$\int_{\Omega} (\sigma^0_{jk,j} + b_k) u_k \, \mathrm{d}\Omega = \int_{\Gamma_2} (p_k - \bar{p}_k) u_k \, \mathrm{d}\Gamma \tag{4.1}$$

in which u_k are the virtual displacements, satisfying the homogeneous boundary conditions $\bar{u}_k \equiv 0$ on Γ_1, b_k the body forces (Fig. 1) and p_k the surface tractions at the point k of the body. (Fig. 3)

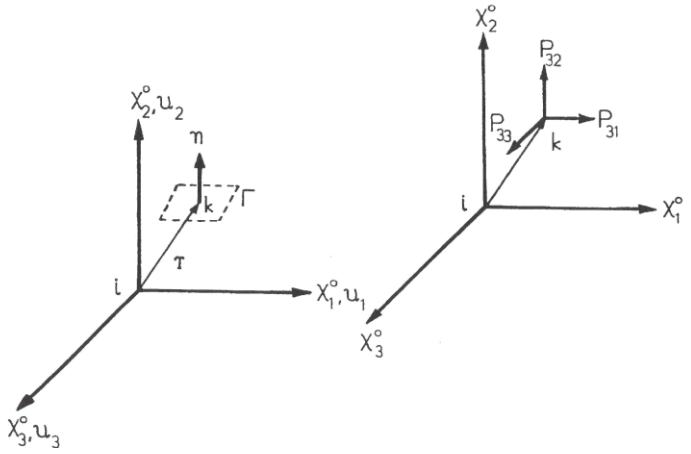

Fig. 3 The stationary system S^0.

Also, (4.1) takes the following form if u_k do not satisfy the previous conditions on Γ_1:

$$\int_{\Omega} (\sigma^0_{jk,j} + b_k) u_k \, \mathrm{d}\Omega = \int_{\Gamma_2} (p_k - \bar{p}_k) u_k \, \mathrm{d}\Gamma + \int_{\Gamma_1} (\bar{u}_k - u_k) p_k \, \mathrm{d}\Gamma \tag{4.2}$$

where $p_k = n_j \sigma^0_{jk}$ are the surface tractions corresponding to the u_k system.

By integrating (4.2) follows:

$$\int_{\Omega} b_k u_k \, \mathrm{d}\Omega - \int_{\Omega} \sigma^0_{jk} \varepsilon_{jk} \, \mathrm{d}\Omega = -\int_{\Gamma_2} \bar{p}_k u_k \, \mathrm{d}\Gamma - \int_{\Gamma_1} p_k u_k \, \mathrm{d}\Gamma + \int_{\Gamma_1} (\bar{u}_k - u_k) p_k \, \mathrm{d}\Gamma \tag{4.3}$$

where ε_{jk} are the strains.

By a second integration (4.3) reduces to:

$$\begin{aligned} &\int_{\Omega} b_k u_k \,\mathrm{d}\Omega + \int_{\Omega} \sigma^0_{jk,j} u_k \,\mathrm{d}\Omega = \\ &-\int_{\Gamma_2} \bar{p}_k u_k \,\mathrm{d}\Gamma - \int_{\Gamma_1} p_k u_k \,\mathrm{d}\Gamma + \int_{\Gamma_1} \bar{u}_k p_k \,\mathrm{d}\Gamma + \int_{\Gamma_2} u_k p_k \,\mathrm{d}\Gamma \end{aligned} \tag{4.4}$$

Beyond the above, a fundamental solution should be found, satisfying the equilibrium equations, of the following type:

$$\sigma^0_{jk,j} + \Delta^i_l = 0 \tag{4.5}$$

where Δ^i_l is the Dirac delta function which represents a unit load at i in the l direction.

The fundamental solution for a three-dimensional isotropic body is: [31]

$$\begin{aligned} u^*_{lk} &= \frac{1}{16\pi G(1-\nu)r}\left[(3-4\nu)\Delta_{lk} + \frac{\partial r}{\partial x_l}\frac{\partial r}{\partial x_k}\right] \\ p^*_{lk} &= -\frac{1}{8\pi(1-\nu)r^2}\left[\frac{\partial r}{\partial n}\left[(1-2\nu)\Delta_{lk} + 3\frac{\partial r}{\partial x_l}\frac{\partial r}{\partial x_k}\right] - \right. \\ &\left. -(1-2\nu)\left[\frac{\partial r}{\partial x_l}n_k - \frac{\partial r}{\partial x_k}n_l\right]\right] \end{aligned} \tag{4.6}$$

where G is the shear modulus, ν Poisson's ratio, n the normal to the surface of the body, Δ_{lk} Kronecker's delta, r the distance from the point of application of the load to the point under consideration and n_j the direction cosines (Fig.3).

The displacements at a point are given by the formula:

$$u^i = \int_{\Gamma} up \,\mathrm{d}\Gamma - \int_{\Gamma} pu \,\mathrm{d}\Gamma + \int_{\Omega} bu \,\mathrm{d}\Omega \tag{4.7}$$

Thus, (4.7) takes the following form for the "l" component:

$$u^i_l = \int_{\Gamma} u_{lk} p_k \,\mathrm{d}\Gamma - \int_{\Gamma} p_{lk} u_k \,\mathrm{d}\Gamma + \int_{\Omega} b_k u_{lk} \,\mathrm{d}\Omega \tag{4.8}$$

By differentiating u at the internal points, one obtains the stress-tensor for an isotropic medium:

$$\sigma^0_{ij} = \frac{2G\nu}{1-2\nu}\Delta_{ij}\frac{\partial u_l}{\partial x_l} + G\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{4.9}$$

Also, after carrying out the differentiation one has:

$$\sigma_{ij}^{0}=\int_{\Gamma}\left[\frac{2Gv}{1-2v}\Delta_{ij}\frac{\vartheta u_{lk}}{\vartheta x_l}+G\left(\frac{\vartheta u_{ik}}{\vartheta x_j}+\frac{\vartheta u_{jk}}{\vartheta x_i}\right)\right]p_k\,\mathrm{d}\Gamma+$$
$$+\int_{\Omega}\left[\frac{2Gv}{1-2v}\Delta_{ij}\frac{\vartheta u_{lk}}{\vartheta x_l}+G\left(\frac{\vartheta u_{ik}}{\vartheta x_j}+\frac{\vartheta u_{jk}}{\vartheta x_i}\right)\right]b_k\,\mathrm{d}\Omega- \tag{4.10}$$
$$-\int_{\Gamma}\left[\frac{2Gv}{1-2v}\Delta_{ij}\frac{\vartheta p_{lk}}{\vartheta x_l}+G\left(\frac{\vartheta p_{ik}}{\vartheta x_j}+\frac{\vartheta p_{jk}}{\vartheta x_i}\right)\right]u_k\,\mathrm{d}\Gamma$$

Eq. (4.10) can be further written as follows:

$$\sigma_{ij}^{0}=\int_{\Gamma}D_{kij}p_k\,\mathrm{d}\Gamma-\int_{\Gamma}S_{kij}u_k\,\mathrm{d}\Gamma+\int_{\Omega}D_{kij}b_k\,\mathrm{d}\Omega \tag{4.11}$$

where the third order tensor components D_{kij} and S_{kij} are:

$$D_{kij}=\frac{1}{8\pi(1-v)r^2}\left[(1-2v)\left[\Delta_{ki}r_{,j}+\Delta_{kj}r_{,i}-\Delta_{ij}r_{,k}\right]+3r_{,i}r_{,j}r_{,k}\right] \tag{4.12}$$

$$S_{kij}=\frac{G}{4\pi(1-v)r^3}\left[3\frac{\vartheta r}{\vartheta n}\left[(1-2v)\Delta_{ij}r_{,k}+v(\Delta_{ik}r_{,j}+\Delta_{jk}r_{,i})-5r_{,i}r_{,j}r_{,k}\right]\right.$$
$$\left.+3v(n_ir_{,j}r_{,k}+n_jr_{,i}r_{,k})+(1-2v)(3n_kr_{,i}r_{,j}+n_j\Delta_{ik}+n_i\Delta_{jk})-(1-4v)n_k\Delta_{ij}\right] \tag{4.13}$$

with: $r_{,i}=\dfrac{\vartheta r}{\vartheta x_i}$

Finally, because of eqs (2.49) and (4.11) by considering the moving system S of Fig. 2, then the stress-tensor reduces to the form:

$$\begin{aligned}
\sigma_{11}&=\sigma_{11}^{0}\\
\sigma_{12}&=\gamma\sigma_{12}^{0}\\
\sigma_{13}&=\gamma\sigma_{13}^{0}\\
\sigma_{21}&=\frac{1}{\gamma}\sigma_{21}^{0}\\
\sigma_{22}&=\sigma_{22}^{0}\\
\sigma_{23}&=\sigma_{23}^{0}\\
\sigma_{31}&=\frac{1}{\gamma}\sigma_{31}^{0}\\
\sigma_{32}&=\sigma_{32}^{0}\\
\sigma_{33}&=\sigma_{33}^{0}
\end{aligned} \tag{4.14}$$

where σ_{ij}^{0} are given by. (4.11) to (4.13).

The following Table 1 shows the values of γ as given by (2.41) for some arbitrary values of the velocity u of the moving aerospace structure:

Table 1

Velocity u	$\gamma = 1/\sqrt{1-u^2/c^2}$	Velocity u	$\gamma = 1/\sqrt{1-u^2/c^2}$
50,000 *km/h*	1.000000001	0.800c	1.666666667
100,000 *km/h*	1.000000004	0.900c	2.294157339
200,000 *km/h*	1.000000017	0.950c	3.202563076
500,000 *km/h*	1.000000107	0.990c	7.088812050
10E+06 *km/h*	1.000000429	0.999c	22.36627204
10E+07 *km/h*	1.000042870	0.9999c	70.71244596
10E+08 *km/h*	1.004314456	0.99999c	223.6073568
2x10E+8 *km/h*	1.017600788	0.999999c	707.1067812
c/3	1.060660172	0.9999999c	2236.067978
c/2	1.154700538	0.99999999c	7071.067812
2c/3	1.341640786	0.999999999c	22360.67978
3c/4	1.511857892	c	∞

From the above Table follows that for small velocities 50,000 *km*/*h* to 200,000 *km*/*h*, the absolute and the relative stress tensor are nearly the same. On the other hand, for bigger velocities like *c*/3, *c*/2 or 3*c*/4 (*c* = speed of light), the variable γ takes values more than the unit and thus, relative stress tensor is very different from the absolute one. Finally, for values of the velocity of the moving structure near the speed of light, the variable γ takes bigger values, while when the velocity is equal to the speed of light, then γ tends to the infinity.

Thus, the Singular Integral Operators Method (S.I.O.M.) as was proposed by E.G.Ladopoulos [4], [8], [9], [11], [12], [13], [15] and E.G.Ladopoulos et al. [22] will be used for the numerical evaluation of the stress tensor (3.11), for every specific case.

5. Conclusions

By the present investigation in the area of aeronautics technologies the theories of *"Relativistic Elasticity"* and *"Relativistic Thermo-Elasticity"* have been investigated and applied for the design of a new generation large aircraft with speeds in the range of 50,000 km/h. Such a design and construction of the new generation aircraft will be applied to an increased market share of International Aeronautical Industries. Furthermore, the theories of *"Relativistic Elasticity"* and *"Relativistic Thermo-Elasticity"* have been applied for the design of the next generation spacecraft moving with very high speeds, even approaching the speed of light, as the target of the International Space Agencies is to achieve such spacecraft in the future. The future investigation concerns to the determination of the proper composite materials or any other kind of materials for the construction of the next generation spacecraft, as usual composite solids are not proper for such a construction.

The theories of *"Relativistic Elasticity"* and *"Relativistic Thermo-Elasticity"* and correspondingly the *"Universal Equation of Elasticity"* and the *"Universal Equation of Therno-Elasticity"* show that there is a considerable difference between the absolute stress tensor of the airframe even in the range of speeds of 50,000 km/h. For bigger speeds the difference between the two stress tensors is very much increased. *"Relativistic Mechanics"* is a combination of the theories of "Classical Elasticity" and "Special Relativity".

For the structural design of the next generation aircraft and spacecraft will be used the stress tensor of the airframe in combination to the singular integral equations. Such a stress tensor is reduced to the solution of a multidimensional singular integral equation and for its numerical evaluation will be used the Singular Integral Operators Method (S.I.O.M.).

References

1. *Ladopoulos E.G*., 'On the numerical solution of the finite – part singular integral equations of the first and the second kind used in fracture mechanics', *Comp. Meth. Appl. Mech. Engng, **65** (1987), 253 – 266.*
2. *Ladopoulos E.G*., 'On the solution of the two – dimensional problem of a plane crack of arbitrary shape in an anisotropic material', *J. Engng Fract. Mech., **28** (1987), 187 – 195.*
3. *Ladopoulos E.G*., 'On the numerical evaluation of the singular integral equations used in two and three-dimensional plasticity problems', *Mech. Res. Commun., **14** (1987), 263 – 274.*
4. *Ladopoulos E.G*., 'Singular integral representation of three – dimensional plasticity fracture problem', *Theor. Appl. Fract. Mech., **8** (1987), 205 – 211.*
5. *Ladopoulos E.G.,* 'On a new integration rule with the Gegenbauer polynomials for singular integral equations, used in the theory of elasticity', *Ing. Arch., **58** (1988), 35 – 46.*
6. *Ladopoulos E.G*., 'On the numerical evaluation of the general type of finite-part singular integrals and integral equations used in fracture mechanics', *J. Engng Fract. Mech., **31** (1988), 315 – 337.*
7. *Ladopoulos E.G*., 'The general type of finite-part singular integrals and integral equations with logarithmic singularities used in fracture mechanics', *Acta Mech.., **75** (1988), 275 – 285.*
8. *Ladopoulos E.G*., 'On the numerical solution of the multidimensional singular integrals and integral equations used in the theory of linear viscoelasticity', *Int J.Math. Math. Scien., **11** (1988), 561 – 574.*
9. *Ladopoulos E.G*., 'Singular integral operators method for two – dimensional plasticity problems', *Comp. Struct., **33** (1989), 859 – 865.*
10. *Ladopoulos E.G*., 'Finite–part singular integro-differential equations arising in two-dimensional aerodynamics', *Arch.. Mech., **41** (1989), 925 – 936.*
11. *Ladopoulos E.G*., 'Cubature formulas for singular integral approximations used in three-dimensional elastiicty', *Rev. Roum. Sci. Tech..,Mec. Appl., **34** (1989), 377 – 389.*
12. *Ladopoulos E.G*., 'Singular integral operators method for three – dimensional elasto – plastic stress analysis', *Comp. Struct., **38** (1991), 1 – 8.*
13. *Ladopoulos E.G*., 'Singular integral operators method for two – dimensional elasto – plastic stress analysis', *Forsch.. Ingen., **57** (1991), 152 – 158.*
14. *Ladopoulos E.G*., 'New aspects for the generalization of the Sokhotski – Plemelj formulae for the solution of finite – part singular integrals used in fracture mechanics', *Int. J. Fract., **54** (1992), 317 – 328.*
15. *Ladopoulos E.G*., 'Singular integral operators method for anisotropic elastic stress analysis', *Comp. Struct., **48** (1993), 965 – 973.*
16. *Ladopoulos E.G*., 'Systems of finite-part singular integral equations in Lp applied to crack problems', *J. Engng Fract. Mech.., **48** (1994), 257 – 266.*
17. *Ladopoulos E.G., Zisis V.A. and Kravvaritis D.,*'Singular integral equations in Hilbert space applied to crack problems', *.Theor.Appl. Fract. Mech., **9** (1988), 271 – 281.*
18. *Zisis V.A. and Ladopoulos E.G*., 'Singular integral approximations in Hilbert spaces for elastic stress analysis in a circular ring with curvilinear cracks', *Indus. Math., **39** (1989), 113 – 134.*
19. *Zisis V.A. and Ladopoulos E.G*., 'Two-dimensional singular inetgral equations exact solutions', *J. Comp. Appl. Math., **31** (1990), 227 – 232.*
20. *Ladopoulos E.G., Kravvaritis D. and Zisis V.A.,*'Finite-part singular integral representation analysis in Lp of two-dimensional elasticity problems', *J. Engng Fract. Mech., **43** (1992), 445 – 454.*
21. *Ladopoulos E.G. and Zisis V.A.,*'Singular integral representation of two-dimensional shear fracture mechanics problem', *Rev.Roum. Sci. Tech., Mec. Appl.., **38** (1993), 617 – 628.*
22. *Ladopoulos E.G., Zisis V.A. and Kravvaritis D.,*'Multidimensional singular integral equations in Lp applied to three-dimensional thermoelastoplastic stress analysis', *Comp. Struct.., **52** (1994), 781 – 788.*
23. *Ladopoulos E.G.,* 'Non-linear integro-differential equations used in orthotropic shallow spherical shell analysis', *Mech. Res. Commun., **18** (1991), 111 – 119.*
24. *Ladopoulos E.G.,* 'Non-linear integro-differential equations in sandwich plates stress analysis', *Mech. Res. Commun., **21** (1994), 95 – 102.*
25. *Ladopoulos E.G.,* 'Non-linear singular integral representation for unsteady inviscid flowfields of 2-D airfoils', *Mech. Res. Commun., **22** (1995), 25 – 34.*
26. *Ladopoulos E.G.,* 'Non-linear multidimensional singular integral equations in 2-dimensional fluid mechanics analysis', *Int. J.Non-Lin. Mech., **35** (2000), 701 – 708.*
27. *Ladopoulos E.G. and Zisis V.A.,*'Existence and uniqueness for non-linear singular integral equations used in fluid mechanics', *Appl. Math., **42** (1997), 345 – 367.*
28. *Ladopoulos E.G. and Zisis V.A.,*'Non-linear finite-part singular integral equations arising in two-dimensional fluid mechanics', *Nonlin. Anal., Th. Meth. Appl., **42** (2000), 277 – 290.*

29. *Ladopoulos E.G. and Zisis V.A.,*'Non-linear singular integral approximations in Banach spaces', *Nonlin. Anal., Th. Meth. Appl.,* ***26*** *(1996), 1293 – 1299.*
30. *Ladopoulos E.G.,* 'Relativistic elastic stress analysis for moving frames', *Rev. Roum. Sci.Tech., Mec. Appl.,* ***36*** *(1991), 195 – 209.*.
31. *Ladopoulos E.G., 'Singular Integral Equations, Linear and Non-Linear Theory and its Applications in Science and Engineering',* Springer Verlag, New York, Berlin, *2000*.
32. *Ladopoulos E.G.,* 'Relativistic mechanics for airframes applied in aeronautical technologies', *Adv. Bound. Elem. Tech..,* ***10*** *(2009), 395 – 405.*
33. *Muskhelishvili N.I.,'Some Basic Problems of the Mathematical Theory of Elasticity',* Noordhoff, Groningen, Netherlands, *1953.*
34. *Green A.E. and Zerna W., 'Theoretical Elasticity',* Oxford Ubniv. Press, Oxford, *1954*.
35. *Boley B.A. and Weiner J.H., 'Theory of Thermal Stresses',* J.Wiley, New York, *1960.*
36. *Nowacki W.,'Thermoelasticity',* Pergamon Press, Oxford, *1962*.
37. *Drucker D.C. and Gilman J.J., 'Fracture of Solids',* J.Wiley, New York, *1963.*
38. *Lekhnitskii S.G., 'Theory of Elasticity of an Anisotropic Elastic Body',* Holden-Day, San Fransisco, *1963.*
39. *Truesdell C. and Noll W., 'The Non-linear Field Theories of Mechanics',* Handbuch der Physic, Vol. III/3, Springer Verlag, Berlin, *1965*.
40. *Liebowitz H. 'Fracture',* Academic Press, New York, *1968.*
41. *Sneddon I.N. and Lowengrub M., 'Crack Problems in the Classical Theory of Elasticity',* J.Wiley, New York, *1969.*
42. *Lions J.L., 'Quelques Methodes de Resolution des Problemes aux Limites Non Lineaires,* Dunod, Paris, *1969.*
43. *Oden J.T., 'Finite Elements in Nonlinear Continua',* McGraw Hill, New York, *1972*.
44. *Eringen A.C., 'Continuum Physics',* Academic Press, New York, *1972.*
45. *Duvant G. and Lions J.L., 'Les Inequations en Mecanique et en Physique', Dunod, Paris,* 1972.
46. *Fichera G., 'Boundary Value Problems of Elasticity with Unilateral Constraints',* Handbuch der Physik, Vol. VIa/2, Springer Verlag, Berlin, *1972*.
47. *Germain P., 'Mecanique des Milieux Continus',* Masson, Paris, *1972.*
48. *Wang C.C. and Truesdell C., 'Introduction to Rational Elasticity',* Noordhoff, Groningen, Netherlands, *1973.*
49. *Washizu K., 'Variational Methods in Elasticity and Plasticity',* Pergamon Press, Oxford, *1975*.
50. *Kupradze V.D., 'Three-dimensional Problems in the Mathematical Theory of Elasticity and Thermoelasticity',* Nauka, Moscow, *1976.*
51. *Gurtin M.E., 'Introduction to Continuum Mechanics',* Academic Press, New York, *1981.*
52. *Ciarlet P.G., 'Topics in Mathematical Elasticity',* North Holland, Amsterdam, *1985.*
53. *Laue M.von, 'Die Relativitätstheorie',* Vol. 1, Vieweg und Sohn, Braunschweig, *1919*.
54. *Gold T., 'Recent Developments in General Relativity',* Pergamon Press, New York, *1962*.
55. *Pirani F.A.E., 'Lectures on General Relativity',* Vol.1, Prentice-Hall, New Jersey, *1964*.
56. *Gursey F., 'Relativity, Groups and Topology',* Gordon and Breach, New York, *1964*.
57. *Adler R., 'Introduction to General Relativity',* McGraw-Hill, New York, *1965*.
58. *Rindler W.*, '*Special Relativity',* Oliver and Boyd, Edinburgh, *1966.*
59. *Möller C., 'The Theory of Relativity',* Oxford University Press, Oxford, *1972*.
60. *Synge J.L., 'General Relativity',* Clarendon Press, Oxford,. *1972*.