*Universal Journal of Aerodynamics* **2 (2014), 1-17**
**www.papersciences.com**

# Special Aerodynamics for Next Generation Aircraft & Spacecraft by Universal Mechanics

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**Abstract**
For the design of the future aircraft with turbojet engines and speeds in the range of 50,000 km/h, or even more and for the future spacecraft of any speed, the new theory of ***"Universal Mechanics"*** is investigated. The proposed theory of *"Universal Mechanics"* consists of the combination of the theories of *"Relativistic Elasticity"* and *"Relativistic Thermo-Elasticity"*. Consequently, according to the above theories there is a considerable difference between the absolute stress tensor and the stress tensor of the airframe even in the range of speeds of 50,000 km/h. In addition, for bigger speeds of the future spacecraft, like c/3, c/2 or 3c/4 (c=speed of light), then the difference between the two stress tensors is very much increased. So, for the future spacecraft with very high speeds, the relative stress tensor will be therefore very much different than the absolute stress tensor. Beyond the above, for velocities near the speed of light, then the values of the relative stress tensor are very much bigger than the corresponding values of the absolute stress tensor. The theory of *"Relativistic Elasticity"* is a combination between the theories of "Classical Elasticity" and "Special Relativity" and results in the ***"Universal Equation of Elasticity"***. Also, the theory of *"Relativistic Thermo-Elasticity"* is a combination between the theories of "Classical Thermo-Elasticity" and "Special Relativity" and results in the ***"Universal Equation of Thermo-Elasticity"***. For the future aircraft and spacecraft of very high speeds special aerodynamics should be used. So, the *"Universal Equation of Elasticity"*and the *"Universal Equation of Thermo-Elasticity"* are parts of the general theory of *"Universal Mechanics"*.

**Key Word and Phrases**
Future Aircraft, Future Spacecraft, Relativistic Elasticity, Relativistic Thermo-Elasticity, Relative Stress Tensor, Absolute Stress Tensor, Stationary and Moving Frames, Energy-Momentum Tensor, Universal Mechanics, Universal Equation of Elasticity, Universal Equation of Thermo-Elasticity.

## 1. Universal Mechanics for Future Aircraft & Spacecraft

It is expected over the next years the big Aeronautical Industries to effect a competitive technological advantage in several strategic areas of new and rapidly developing advanced technologies. So, this considerably big market share includes the design of a future aircraft with speeds even in the range of 50,000 km/h. Hence, as the design of next generation turbojet engines makes possible the design of such type of large aircraft, then there is a need of elastic stress design and analysis for the construction of the total parts of such type of the future and next generation aircraft.

Beyond the above, the scope by the big Space Agencies is to achieve in the future, an absolute spacecraft moving with very high speeds, even approaching the speed of light. So, how far could be this future ? According to the current investigation and research such future could be much closer than everybody believes. For the future spacecraft the relative stress tensor will be much different than the absolute stress tensor and so special solid should be used for the construction of the next generation spacecraft.

Hence, the suitable choice of the solid which could be used for the construction of the absolute spacecraft is under investigation, but such solid will be very much different than the usual composite materials.

It will be therefore shown that there is a significant difference between the absolute stress tensor and the stress tensor of the airframe even in the range of speeds of 50,000 km/h. In addition, for bigger speeds the difference of the two stress tensors will be very much increased. So, for bigger velocities like c/3, c/2 or 3c/4 (c=speed of light) the relative stress tensor is very much different than the absolute one and for velocities near the speed of light the values of the relative stress tensor are much bigger than the corresponding values of the absolute stress tensor. The study of the connection between the stress tensors of the absolute frame and the airframe is included in the theory proposed by E.G.Ladopoulos [30] - [32] under the term *"Relativistic Elasticity"* and *"Relativistic Thermo-Elasticity"* and the final formula which results from the above theories is called the ***"Universal Equation of Elasticity"*** and the***"Universal Equation of Thermo-Elasticity"***, correspondingly. Furthermore, both theories of *"Relativistic Elasticity"* and *"Relativistic Thermo-Elasticity"* are included in a more general theory under the term ***"Universal Mechanics"***.

Moreover, E.G.Ladopoulos [1]-[16] and E.G.Ladopoulos et al. [17]-[22] proposed singular integral equation methods applied to elasticity, plasticity and fracture mechanics theories. In the above mentioned papers the ***Singular Integral Operators Method (S.I.O.M.)*** is proposed for the numerical solution of the multidimensional singular integral equations in which the stress tensor analysis of the linear elastic theory is reduced. Beyond the above, the theory of linear singular integral equations was extended to non-linear singular integral equations, too. [23]-[29]. So, the theory of *"Universal Mechanics"* and correspondingly the theories of *"Relativistic Elasticity"* and *"Relativistic Thermo-Elasticity"* will be applied for the design of the elastic stress analysis of the airframes.

In addition, the classical theory of elastic stress analysis and thermo-elastic stress analysis began to be analyzed in the early nineteenth century and was further developed during the twentieth century. In the past, several important monographs were published on the classical theory of elasticity and thermo-elasticity. [33]-[52].

On the other hand, over the past years special attention has been given, by many scientists worldwide, on the theoretical aspects of the special theory of relativity. So, some classical monographs were written, dealing with the theoretical foundations and investigations of the special and the general theory of relativity. [53]–[60]. Also, by the current investigation will be shown that the **"*relative stress tensor is not symmetrical*"**, while, as it is well known, the **"*absolute stress tensor is symmetrical*"**. Such a difference is very important for the design of the future aircraft and spacecraft of very high speeds.

Finally, for the future aircraft and spacecraft of very high speeds special aerodynamics will be used.

## 2. Relativistic Elasticity - Universal Equation of Elasticity for Future Aircraft & Spacecraft

Let us consider the state of stress at a point in the stationary frame $S^0$, defined by the following symmetrical stress tensor: (Fig.1)

$$\sigma^0 = \begin{bmatrix} \sigma_{11}^0 & \sigma_{12}^0 & \sigma_{13}^0 \\ \sigma_{21}^0 & \sigma_{22}^0 & \sigma_{23}^0 \\ \sigma_{31}^0 & \sigma_{32}^0 & \sigma_{33}^0 \end{bmatrix} \tag{2.1}$$

where: $$\sigma_{21}^0 = \sigma_{12}^0, \sigma_{31}^0 = \sigma_{13}^0, \sigma_{32}^0 = \sigma_{23}^0 \tag{2.2}$$

Beyond the above, consider an infinitesimal face element $df$ with a directed normal, defined by a unit vector **n**, at definite point $p$ in the three-space of a Lorenz system. The matter on either side of this face element experiences a force which is proportional to $df$.

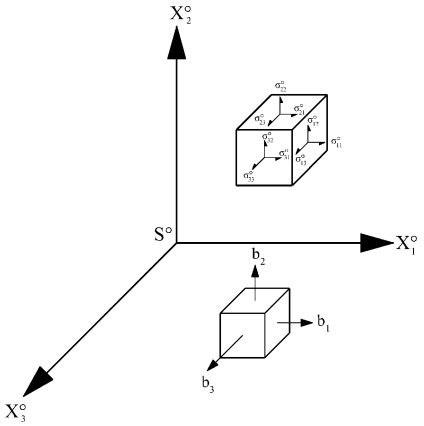


**Fig. 1** The state of stress $\sigma_{ik}^{0}$ in the stationary system $S^{0}$.

Hence, the force is valid as:

$$\mathrm{d}\,\boldsymbol{\sigma}(\mathbf{n}) = \boldsymbol{\sigma}(\mathbf{n})\,\mathrm{d}\,f \tag{2.3}$$

The components $\sigma_i(\mathbf{n})$ of $\boldsymbol{\sigma}(\mathbf{n})$ are linear functions of the components $n_k$ of $\mathbf{n}$:

$$\sigma_i(\mathbf{n}) = \sigma_{ik} n_k\,,\ i,k = 1,2,3 \tag{2.4}$$

where $\sigma_{ik}$ is the elastic stress tensor, also called as the relative stress tensor, in contrast to the space part $\sigma_{ik}^{0}$ of the total energy-momentum tensor $T_{ik}$, referred as the absolute stress tensor. [53], [54} (Fig. 2).

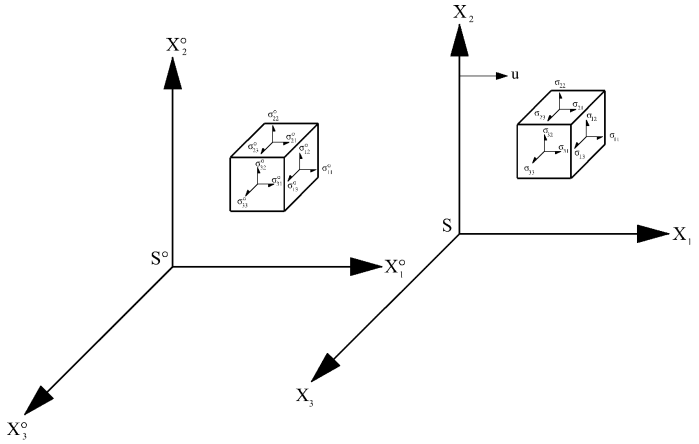


**Fig. 2** The state of stress $\sigma_{ik}^{0}$ in the stationary system $S^{0}$ and $\sigma_{ik}$ in the airframe system with velocity $u$ parallel to the $x_1$ - axis.

Also, the connection between the absolute and relative stress tensors is defined as:

$$\sigma_{ik}^{0} = \sigma_{ik} + g_i u_k,\ i,k = 1,2,3 \tag{2.5}$$

in which $g_i$ are the components of the momentum density $\mathbf{g}$ and $u_k$ the components of the velocity $\mathbf{u}$ of the matter.

The connection between $\mathbf{g}$ and the energy flux $\mathbf{s}$, is equal to:

$$\mathbf{g} = \mathbf{s}/c^2 \tag{2.6}$$

where $c$ denotes the speed of light (= 300.000 *km*/*sec*).

Moreover, the total work done per unit time by elastic forces on the matter inside the closed surface $f$ can be given by the formula:

$$W = \int_f (\boldsymbol{\sigma}(\mathbf{n}) \cdot \mathbf{u}) \mathrm{d}f = \int_f \sigma_{ik} n_k u_i \,\mathrm{d}f = -\int_\upsilon \frac{\vartheta(u_i \sigma_{ik})}{\vartheta x_k} \mathrm{d}\upsilon,\ i,k = 1,2,3 \tag{2.7}$$

where the integration in the last integral is extended over the interior $\upsilon$ of the surface $f$.

Consequently, the work done on an infinitesimal piece of matter of volume $\delta\upsilon$ is valid as:

$$\delta W = -\frac{\vartheta(u_i \sigma_{ik})}{\vartheta x_k} \delta\upsilon \tag{2.8}$$

Furthermore, (2.8) must be equal to the increase per unit time of the energy inside $\delta\upsilon$:

$$\frac{\mathrm{d}}{\mathrm{d}t}(h\delta\upsilon) = \delta W \tag{2.9}$$

in which $h$ denotes the total energy density, including the elastic energy and $\mathrm{d}/\mathrm{d}t$ is the substantial time derivative.

Eq. (2.9) is valid as:

$$\frac{\mathrm{d}}{\mathrm{d}t}(h\delta\upsilon) = \left(\frac{\vartheta h}{\vartheta t} + \frac{\vartheta h}{\vartheta x_k} u_k\right)\delta\upsilon + h\delta\upsilon \frac{\vartheta u_k}{\vartheta x_k} = \left[\frac{\vartheta h}{\vartheta t} + \frac{\vartheta}{\vartheta x_k}(h u_k)\right]\delta\upsilon \tag{2.10}$$

which finally leads to the relation:

$$\frac{\vartheta h}{\vartheta t} + \frac{\vartheta}{\vartheta x_k}(h u_k + u_i \sigma_{ik}) = 0 \tag{2.11}$$

So, the total energy flow is valid as:

$$\mathbf{s} = \mathbf{h}\mathbf{u} + (\mathbf{u} \cdot \boldsymbol{\sigma}) \tag{2.12}$$

where $(\mathbf{u} \cdot \boldsymbol{\sigma})$ is a space vector with components $(\mathbf{u} \cdot \boldsymbol{\sigma})_k = u_i \sigma_{ik}$.

Hence, the total momentum density can be written as:

$$\mathbf{g} = \frac{\mathbf{s}}{c^2} = \mu\mathbf{u} + \frac{(\mathbf{u}\cdot\boldsymbol{\sigma})}{c^2} \tag{2.13}$$

in which $\mu = h/c^2$ denotes the total mass density, including the mass of the elastic energy.

From (2.5) and (2.13) one obtains:

$$\sigma_{ik} - \sigma_{ki} = -g_i u_k + g_k u_i = [-(\mathbf{u}\cdot\boldsymbol{\sigma})_i u_k + (\mathbf{u}\cdot\boldsymbol{\sigma})_k u_i]/c^2 \neq 0 \tag{2.14}$$

which shows that the relative stress tensor is not symmetrical, in contrast to the absolute stress tensor (2.1) which is symmetrical.

In the stationary frame $S^0$ the velocity $u^0 = 0$ and so, from (2.5), (2.12) and (2.13) the following expressions are obtained:

$$\sigma^0_{ik} = \sigma_{ik} = \sigma_{ki} = \sigma^0_{ki} \ (i,k = 1,2,3) \tag{2.15}$$

In addition, the mechanical energy-momentum tensor satisfies the following relation:

$$T_{ik}U_k = -h^0 U_i \tag{2.16}$$

where $U_i$ denotes the four-velocity of the matter, in the Lorentz system and $U^0_i = (0,0,0,ic)$.

Consequently, the following scalar can be formed:

$$U_i T_{ik} U_k / c^2 = U^0_i T^0_{ik} U^0_k / c^2 = -T^0_{44} = h^0(x_1) \tag{2.17}$$

with $h^0(x_1)$ the invariant rest energy density considered as a scalar function of the coordinates ($x_i$) ($i$ = 1,2,3) in $S$. (Fig. 2)

Beyond the above, by applying the tensor:

$$\Delta_{ik} = \delta_{ik} + U_i U_k / c^2 \tag{2.18}$$

which satisfies the relations:

$$U_i \Delta_{ik} = \Delta_{ik} U_k = 0 \tag{2.19}$$

then, the following symmetrical tensor can be formed:

$$S_{ik} = \Delta_{i1} T_{1m} \Delta_{mk} = S_{ki} \tag{2.20}$$

which is orthogonal to $U_i$:

$$U_i S_{ik} = S_{ik} U_k = 0 \tag{2.21}$$

By combining eqs. (2.16), (2.17) and (2.20) we have:

$$S_{ik} = T_{ik} - h^0 U_i U_k / c^2 \tag{2.22}$$

In addition, in the stationary system $S_0$ one has:

$$S_{ik}^0 = \sigma_{ik}^0 = \sigma_{ik},\ S_{i4}^0 = S_{4i}^0 = 0 \tag{2.23}$$

Eq. (2.22) may also be written as:

$$T_{ik} = \xi_{ik} + S_{ik} \tag{2.24}$$

in which:

$$\xi_{ik} = h^0 U_i U_k / c^2 = \mu^0 U_i U_k \tag{2.25}$$

is the kinetic energy-momentum tensor for an elastic body and:

$$\mu^0 = h^0 / c^2 \tag{2.26}$$

is the proper mass density.

We introduce further in every system $S$ the quantity:

$$\sigma_{ik} = S_{ik} - S_{i4} U_k / U_4 \tag{2.27}$$

which, on account of (2.24) and (2.25) is valid as:

$$\sigma_{ik} = T_{ik} - T_{i4} U_k / U_4 \tag{2.28}$$

From (2.1) and (2.2) the three-tensor:

$$S_{ik}^0 = \sigma_{ik}^0 = \sigma_{ik}$$

in the stationary system is a real symmetrical matrix. The corresponding normalized eigenvectors $\mathbf{h}^{0(j)}$ satisfy the orthonormality relations:

$$\mathbf{h}^{(j)0} \cdot \mathbf{h}^{(\rho)0} = \delta^{je} \tag{2.29a}$$

and:

$$h_i^{(j)0} h_k^{(j)0} = \delta_{ik} \quad (j, \rho = 1,2,3) \tag{2.29b}$$

The eigenvalues $p_{(j)}^0$, the principal stresses, are the three roots of the following algebraic equation, where $\lambda$ is the unknown:

$$\left| S_{ik}^0 - \lambda \delta_{ik} \right| = \left| \sigma_{ik}^0 - \lambda \delta_{ik} \right| = 0 \tag{2.30}$$

The matrix $S_{ik}^0$ can be further written in terms of the eigenvalues and eigenvectors as:

$$S_{ik}^0 = \sigma_{ik}^0 = p_{(j)}^0 h_i^{(j)0} h_k^{(j)0} \tag{2.31}$$

Then, from eqs. (2.23) and (2.31) we obtain the following form of the stress four-tensor in $S^o$:

$$S_{ik}^0 = p_{(j)}^0 h_i^{(j)0} h_k^{(j)0} \tag{2.32}$$

Hence, in any system $S$ one has:

$$S_{ik} = p_{(j)}^0 h_i^{(j)} h_k^{(j)} \tag{2.33}$$

From (2.24), (2.25), (2.27) and (2.33) follow the expressions:

$$T_{ik} = \mu^0 U_i U_k + p_{(j)}^0 h_i^{(j)} h_k^{(j)} \tag{2.34}$$

$$\sigma_{ik} = S_{ik} - S_{i4} U_k / U_4 = p_{(j)}^0 h_k^{(j)} \left( h_k^{(j)} + i h_4^{(j)} u_k / c \right) \tag{2.35}$$

By putting:

$$h_i^{(j)} = (\mathbf{h}^{(j)}, h_4^{(j)}) \tag{2.36}$$

and introducing the notation $\mathbf{a} \bullet \mathbf{b}$ for the direct product of the vectors $\mathbf{a}$ and $\mathbf{b}$, then eqn (2.35) can be written for the relative stress tensor $\boldsymbol{\sigma}$ as following:

$$\boldsymbol{\sigma} = p_{(j)}^0 \left[ \mathbf{h}^{(j)} \bullet \mathbf{h}^{(j)} + \frac{i}{c} h_4^{(j)} (\mathbf{h}^{(j)} \bullet \mathbf{u}) \right], j = 1,2,3 \tag{2.37}$$

Furthermore, the triad vectors $h_i^{(j)}$ satisfy the tensor relations:

$$h_i^{(j)} h_i^{(\rho)} = \delta^{j\rho} \tag{2.38}$$

$$h_i^{(j)} h_k^{(j)} = \Delta_{ik} \tag{2.39}$$

with $\Delta_{ik}$ given by (2.18).

If the stationary system $S^0$ for every event point is chosen in such a way that the spatial axes in $S^0$ and in $S$ have the same orientation, one has:

$$\begin{aligned} \mathbf{h}^{(j)} &= \mathbf{h}^{(j)0} + \left\{ \mathbf{u} (\mathbf{u} \cdot \mathbf{h}^{(j)0}) (\gamma - 1) \right\} / u^2 \\ h_4^{(j)} &= i \mathbf{u} \cdot \mathbf{h}^{(j)0} \gamma / c \end{aligned} \tag{2.40}$$

with:

$$\gamma = 1 / (1 - u^2 / c^2)^{1/2} \tag{2.41}$$

From (2.34) and (2.40) with $i = k = 4$ follows:

$$h = -T_{44} = -\mu^0 U_4^2 - p_{(j)}^0 (\mathbf{u} \cdot \mathbf{h}^{(j)0})^2 \cdot \gamma^2 / c^2 \tag{2.42}$$

In the stationary system, (2.37) reduces to:

$$\boldsymbol{\sigma}^0 = p^0_{(j)}\left(\mathbf{h}^{(j)0} \bullet \mathbf{h}^{(j)0}\right) \tag{2.43}$$

Thus, from (2.42) we have the following transformation law for the energy density:

$$h = \frac{h^0 + \mathbf{u}\cdot\boldsymbol{\sigma}^0\cdot\mathbf{u}/c^2}{1-u^2/c^2}$$
$$\mathbf{u}\cdot\boldsymbol{\sigma}^0\cdot\mathbf{u} = u_i\sigma^0_{ik}u_k \tag{2.44}$$

and the mass density:

$$\mu = \frac{\mu^0 + \mathbf{u}\cdot\boldsymbol{\sigma}^0\cdot\mathbf{u}/c^4}{1-u^2/c^2} \tag{2.45}$$

From (2.40) and (2.34) with $k$ = 4, we obtain the momentum density $\mathbf{g}$ with the components $g_i = T_{i4}/ic$:

$$\mathbf{g} = \mathbf{u}\left[h^0 + \mathbf{u}\cdot\boldsymbol{\sigma}^0\cdot\mathbf{u}(1-\gamma^{-1})/u^2\right]\gamma^2/c^2 + (\boldsymbol{\sigma}^0\cdot\mathbf{u})\gamma/c^2$$
$$(\boldsymbol{\sigma}^0\cdot\mathbf{u})_i = \sigma^0_{ik}u_k \tag{2.46}$$

Beyond the above, from (2.40) and (2.35) one has the relative stress tensor:

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}^0 + \mathbf{u}\bullet(\boldsymbol{\sigma}^0\cdot\mathbf{u})(\gamma-1)/u^2 - (\boldsymbol{\sigma}^0\cdot\mathbf{u})\bullet\mathbf{u}(\gamma-1)/\gamma u^2$$
$$-(\mathbf{u}\bullet\mathbf{u})(\mathbf{u}\cdot\boldsymbol{\sigma}^0\cdot\mathbf{u})(\gamma-1)^2/\gamma u^4 \tag{2.47}$$

In the special case $\mathbf{u}$ = ($u$,0,0), where the notation of the matter at the point considered is parallel to the $x_1$-axis (see Figs.1 and 2), the transformation equations (2.44), (2.46) and (2.47) reduce to:

$$h = \left(h^0 + \frac{u^2}{c^2}\sigma^0_{11}\right)\gamma^2$$
$$g_{x_1} = \gamma^2\left(\mu^0 + \frac{\sigma^0_{11}}{c^2}\right)u$$
$$g_{x_2} = \frac{\gamma\sigma^0_{21}}{c^2}u \tag{2.48}$$
$$g_{x_3} = \frac{\gamma\sigma^0_{31}}{c^2}u$$

and the relative stress tensor gives the ***Universal Equation of Elasticity***:

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} = \begin{bmatrix} \sigma_{11}^0 & \gamma\sigma_{12}^0 & \gamma\sigma_{13}^0 \\ \frac{1}{\gamma}\sigma_{21}^0 & \sigma_{22}^0 & \sigma_{23}^0 \\ \frac{1}{\gamma}\sigma_{31}^0 & \sigma_{32}^0 & \sigma_{33}^0 \end{bmatrix} \tag{2.49}$$

in which $\gamma$ is given by (2.41). Finally, as it could be easily seen the relative stress tensor is not symmetrical, in contrast to the absolute stress tensor which is symmetrical.

## 3. Relativistic Thermo-Elasticity - Universal Equation of Thermo-Elasticity for Future Aircraft & Spacecraft

In the previous sections the system under investigation, which is the elastic body, was regarded as a purely mechanical system. However, all macroscopic systems are in reality thermo-dynamical systems with properties depending on non-mechanical variables such as the proper temperature $T^o$, and so the question which arises is to what kind of thermodynamical processes may be described by an energy-momentum tensor.

So, it is clear that all properties in which heat energy is transferred from one part of the system to another are excluded, for heat flow in the manner would give rise to a non-vanishing energy current in the rest system.

Let us consider further a general system of continuously distributed ponderable or visible matter, inside which invisible heat conduction can take place, while the motion of the visible matter is described by the four-velocity $U_i$. Then the energy-momentum tensor of the general system can be given by the following relation:

$$T_{ik} = M_{ik} + H_{ik} \tag{3.1}$$

where $M_{ik}$ is the mechanical part of the energy-momentum tensor and $H_{ik}$ the heat part.

Furthermore, the mechanical part $M_{ik}$ is valid by the following formula:

$$M_{ik} = d^0 U_i U_k / c^2 + S_{ik} \tag{3.2}$$

and the heat part:

$$H_{ik} = (U_i V_k + V_i U_k)/c^2 \tag{3.3}$$

where the four-vector $V_i$ satisfies the relation:

$$V_i = -\Delta_{ik} T_{kj} U_j = -T_{ik} U_k - d^0 U_i \tag{3.4}$$

in which $d^0$ denote the normalized eigenvectors, $\Delta_{ik}$ is the tensor given by (2.18) and $P_{ik}$ the potential part of the energy momentum tensor.

The four-vector $V_i$ is orthogonal to $U_i$:

$$U_i V_i = 0 \tag{3.5}$$

and so we obtain:

$$V_i = \left(\mathbf{V}, i(\mathbf{V},\mathbf{u})/c\right) \tag{3.6}$$

where $\mathbf{u}$ denotes the velocity of the matter.

Consequently, in the stationary system, (3.6) reduces to:

$$V_i^0 = \left(\mathbf{V}^0, 0\right) \tag{3.7}$$

In addition, by replacing (2.18) into (2.20) and using (2.17) and (3.4), then we have instead of (2.22):

$$S_{ik} = T_{ik} - d^0 U_i U_k / c^2 - \left(U_i V_k + V_i U_k\right)/c^2 \tag{3.8}$$

So, from (3.8) follows the required relation (3.1), instead of (2.24).

Consider further the general system of continuously matter described previously inside which invisible heat conduction can take place, while the motion of the matter is described by the four-velocity $U_i$ or by the velocity $u_i$.

Then, for the connection between the energy-momentum tensor $T_{ik}$ and the relative stress tensor $\sigma_{ik}$ of the general system, the following relation is valid:

$$T_{ik} = g_i u_k + \sigma_{ik} + u_i \xi_k / c^2 \tag{3.9}$$

with:

$$\xi_k = U_4 \left(V_k - V_4 U_k / U_4\right)/ic \tag{3.10}$$

in which $V_k$ denotes the four-vector given by (3.4), $g_i$ the momentum density and $c$ the speed of light.

The quantity $\xi_k$ seems to be the most important part of $\xi_{ik}$:

$$\xi_{ik} = H_{ik} - H_{i4} U_k / U_4 = U_i \left(V_k - V_4 U_k / U_4\right)/c^2 \tag{3.11}$$

Also, $\xi_k$ can be written by the following form by using (2.41) and (3.6):

$$\xi_k = \left(\boldsymbol{\xi}, 0\right) \tag{3.12}$$

with:

$$\boldsymbol{\xi} = \gamma\left[\mathbf{V} - \mathbf{u}(\mathbf{V},\mathbf{u})/c^2\right] \tag{3.13}$$

In the stationary system, $\boldsymbol{\xi}^0$ is equal to the heat current density $\mathbf{V}^0$:

$$\boldsymbol{\xi}^0 = \mathbf{V}^0 \tag{3.14}$$

By combining (3.10) and (3.11), then we have:

$$\xi_{ik} = U_i \, \xi_k \big/ \gamma c^2 \tag{3.15}$$

Thus, by using (2.35), (3.1), (3.2), (3.11) and (3.15), one obtains:

$$T_{ik} - T_{i4} \, U_k / U_4 = \sigma_{ik} + \xi_{ik} = \sigma_{ik} + U_i \, \xi_k \big/ \gamma c^2 \tag{3.16}$$

which finally reduces to the required formula (3.9).

Beyond the above, let us consider the general system of continuously matter, inside which invisible heat conduction can take place. Then the momentum density $\mathbf{g}$ of this system is given by the ***Universal Equation of Thermo-Elasticity:***

$$\mathbf{g} = m\mathbf{u} + \frac{(\mathbf{u}, \boldsymbol{\sigma})}{c^2} + \frac{\boldsymbol{\xi}}{c^2} \tag{3.17}$$

where $\mathbf{u}$ denotes the velocity of the matter at the place and time considered, $\boldsymbol{\sigma}$ the relative stress tensor, $\boldsymbol{\xi}$ is given by (3.13) and $m = E/c^2$ is the total mass density.

From (3.9), we obtain for the energy current density:

$$D_k = E u_k + u_i \sigma_{ik} + \xi_k \tag{3.18}$$

which can be further written as:

$$\mathbf{D} = E\mathbf{u} + (\mathbf{u}, \boldsymbol{\sigma}) + \boldsymbol{\xi} \tag{3.19}$$

So, from (3.19) by using the formula of the momentum density $\mathbf{g}$:

$$\mathbf{g} = \mathbf{D} \big/ c^2 \tag{3.20}$$

we obtain the required relation (3.17) which is a generalization, for a general system with heat conduction.

## 4. Universal Mechanics by Elastic Stress Analysis for Future Aircraft & Spacecraft

Let us consider the stationary frame of Fig. 1 with $\Gamma_1$ the portion of the boundary of the body on which displacements are presented, $\Gamma_2$ the surface of the body on which the force tractions are employed and $\Gamma$ the total surface of the body equal to $\Gamma_1 + \Gamma_2$.

Also, for the principal of virtual displacements, for linear elastic problems then the following formula is valid:

$$\int_{\Omega} (\sigma^0_{jk,j} + b_k) u_k \, \mathrm{d}\Omega = \int_{\Gamma_2} (p_k - \bar{p}_k) u_k \, \mathrm{d}\Gamma \tag{4.1}$$

where $u_k$ are the virtual displacements, satisfying the homogeneous boundary conditions $\bar{u}_k \equiv 0$ on $\Gamma_1$, $b_k$ the body forces (Fig. 1) and $p_k$ the surface tractions at the point $k$ of the body. (Fig. 3)

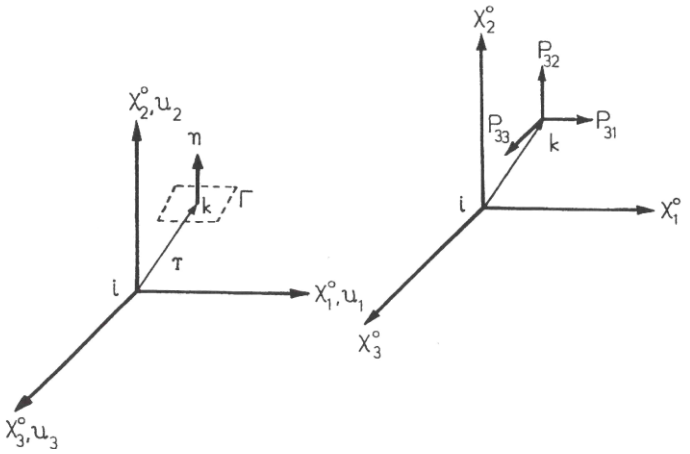


**Fig. 3** The stationary system $S^0$.

Eqn (4.1) can be further written as following if $u_k$ do not satisfy the previous conditions on $\Gamma_1$:

$$\int_{\Omega} (\sigma^0_{jk,j} + b_k) u_k \, \mathrm{d}\Omega = \int_{\Gamma_2} (p_k - \bar{p}_k) u_k \, \mathrm{d}\Gamma + \int_{\Gamma_1} (\bar{u}_k - u_k) p_k \, \mathrm{d}\Gamma \tag{4.2}$$

where $p_k = n_j \sigma^0_{jk}$ are the surface tractions corresponding to the $u_k$ system.

Then, by integrating (4.2) follows:

$$\int_{\Omega} b_k u_k \, \mathrm{d}\Omega - \int_{\Omega} \sigma^0_{jk} \varepsilon_{jk} \, \mathrm{d}\Omega = -\int_{\Gamma_2} \bar{p}_k u_k \, \mathrm{d}\Gamma - \int_{\Gamma_1} p_k u_k \, \mathrm{d}\Gamma + \int_{\Gamma_1} (\bar{u}_k - u_k) p_k \, \mathrm{d}\Gamma \tag{4.3}$$

in which $\varepsilon_{jk}$ are the strains.

By a second integration then (4.3) reduces to:

$$\begin{aligned} &\int_{\Omega} b_k u_k \, \mathrm{d}\Omega + \int_{\Omega} \sigma^0_{jk,j} u_k \, \mathrm{d}\Omega = \\ &- \int_{\Gamma_2} \bar{p}_k u_k \, \mathrm{d}\Gamma - \int_{\Gamma_1} p_k u_k \, \mathrm{d}\Gamma + \int_{\Gamma_1} \bar{u}_k p_k \, \mathrm{d}\Gamma + \int_{\Gamma_2} u_k p_k \, \mathrm{d}\Gamma \end{aligned} \tag{4.4}$$

Beyond the above, a fundamental solution should be found, satisfying the equilibrium equations, of the following type:

$$\sigma^0_{jk,j} + \Delta^i_l = 0 \tag{4.5}$$

where $\Delta^i_l$ denotes the Dirac delta function which represents a unit load at $i$ in the $l$ direction.

The fundamental solution for a three-dimensional isotropic body is: [31]

$$u_{lk}^{*} = \frac{1}{16\pi G(1-v)r}\left[(3-4v)\Delta_{lk} + \frac{\vartheta r}{\vartheta x_l}\frac{\vartheta r}{\vartheta x_k}\right]$$

$$p_{lk}^{*} = -\frac{1}{8\pi(1-v)r^2}\left[\frac{\vartheta r}{\vartheta n}\left[(1-2v)\Delta_{lk} + 3\frac{\vartheta r}{\vartheta x_l}\frac{\vartheta r}{\vartheta x_k}\right] - \right. \tag{4.6}$$

$$\left. -(1-2v)\left[\frac{\vartheta r}{\vartheta x_l}n_k - \frac{\vartheta r}{\vartheta x_k}n_l\right]\right]$$

where $G$ is the shear modulus, $v$ Poisson's ratio, $n$ the normal to the surface of the body, $\Delta_{lk}$ Kronecker's delta, $r$ the distance from the point of application of the load to the point under consideration and $n_j$ the direction cosines (Fig.3).

The displacements at a point are given as following:

$$u^i = \int_{\Gamma} up\,\mathrm{d}\Gamma - \int_{\Gamma} pu\,\mathrm{d}\Gamma + \int_{\Omega} bu\,\mathrm{d}\Omega \tag{4.7}$$

Hence, (4.7) takes the following form for the "$l$" component:

$$u_l^i = \int_{\Gamma} u_{lk}p_k\,\mathrm{d}\Gamma - \int_{\Gamma} p_{lk}u_k\,\mathrm{d}\Gamma + \int_{\Omega} b_k u_{lk}\,\mathrm{d}\Omega \tag{4.8}$$

By differentiating $u$ at the internal points, one obtains the stress-tensor for an isotropic medium:

$$\sigma_{ij}^0 = \frac{2Gv}{1-2v}\Delta_{ij}\frac{\vartheta u_l}{\vartheta x_l} + G\left(\frac{\vartheta u_i}{\vartheta x_j} + \frac{\vartheta u_j}{\vartheta x_i}\right) \tag{4.9}$$

Moreover, after carrying out the differentiation one has:

$$\sigma_{ij}^0 = \int_{\Gamma}\left[\frac{2Gv}{1-2v}\Delta_{ij}\frac{\vartheta u_{lk}}{\vartheta x_l} + G\left(\frac{\vartheta u_{ik}}{\vartheta x_j} + \frac{\vartheta u_{jk}}{\vartheta x_i}\right)\right]p_k\,\mathrm{d}\Gamma +$$

$$+\int_{\Omega}\left[\frac{2Gv}{1-2v}\Delta_{ij}\frac{\vartheta u_{lk}}{\vartheta x_l} + G\left(\frac{\vartheta u_{ik}}{\vartheta x_j} + \frac{\vartheta u_{jk}}{\vartheta x_i}\right)\right]b_k\,\mathrm{d}\Omega - \tag{4.10}$$

$$-\int_{\Gamma}\left[\frac{2Gv}{1-2v}\Delta_{ij}\frac{\vartheta p_{lk}}{\vartheta x_l} + G\left(\frac{\vartheta p_{ik}}{\vartheta x_j} + \frac{\vartheta p_{jk}}{\vartheta x_i}\right)\right]u_k\,\mathrm{d}\Gamma$$

Eq. (4.10) can be further written as follows:

$$\sigma_{ij}^0 = \int_{\Gamma} D_{kij}p_k\,\mathrm{d}\Gamma - \int_{\Gamma} S_{kij}u_k\,\mathrm{d}\Gamma + \int_{\Omega} D_{kij}b_k\,\mathrm{d}\Omega \tag{4.11}$$

in which the third order tensor components $D_{kij}$ and $S_{kij}$ are:

$$D_{kij} = \frac{1}{8\pi(1-\nu)r^2}\left[(1-2\nu)\left[\Delta_{ki}r_{,j} + \Delta_{kj}r_{,i} - \Delta_{ij}r_{,k}\right] + 3r_{,i}r_{,j}r_{,k}\right] \tag{4.12}$$

$$S_{kij} = \frac{G}{4\pi(1-\nu)r^3}\left[3\frac{\vartheta r}{\vartheta n}\left[(1-2\nu)\Delta_{ij}r_{,k} + \nu(\Delta_{ik}r_{,j} + \Delta_{jk}r_{,i}) - 5r_{,i}r_{,j}r_{,k}\right]\right.$$
$$\left. + 3\nu(n_i r_{,j}r_{,k} + n_j r_{,i}r_{,k}) + (1-2\nu)(3n_k r_{,i}r_{,j} + n_j\Delta_{ik} + n_i\Delta_{jk}) - (1-4\nu)n_k\Delta_{ij}\right] \tag{4.13}$$

with: $r_{,i} = \dfrac{\vartheta r}{\vartheta x_i}$

Finally, because of eqs (2.49) and (4.11) by considering the moving system $S$ of Fig. 2, then the stress-tensor reduces to the following form:

$$\begin{aligned}
\sigma_{11} &= \sigma_{11}^0 \\
\sigma_{12} &= \gamma\sigma_{12}^0 \\
\sigma_{13} &= \gamma\sigma_{13}^0 \\
\sigma_{21} &= \frac{1}{\gamma}\sigma_{21}^0 \\
\sigma_{22} &= \sigma_{22}^0 \\
\sigma_{23} &= \sigma_{23}^0 \\
\sigma_{31} &= \frac{1}{\gamma}\sigma_{31}^0 \\
\sigma_{32} &= \sigma_{32}^0 \\
\sigma_{33} &= \sigma_{33}^0
\end{aligned} \tag{4.14}$$

where $\sigma_{ij}^0$ are given by. (4.11) to (4.13).

Table 1 shows the values of $\gamma$ as given by (2.41) for some arbitrary values of the velocity $u$ of the moving aerospace structure:

**Table 1**

| Velocity $u$ | $\gamma = 1/\sqrt{1-u^2/c^2}$ | Velocity $u$ | $\gamma = 1/\sqrt{1-u^2/c^2}$ |
|---|---|---|---|
| **50,000 *km/h*** | **1.000000001** | **0.800c** | **1.666666667** |
| **100,000 *km/h*** | **1.000000004** | **0.900c** | **2.294157339** |
| **200,000 *km/h*** | **1.000000017** | **0.950c** | **3.202563076** |
| **500,000 *km/h*** | **1.000000107** | **0.990c** | **7.088812050** |
| **10E+06 *km/h*** | **1.000000429** | **0.999c** | **22.36627204** |
| **10E+07 *km/h*** | **1.000042870** | **0.9999c** | **70.71244596** |
| **10E+08 *km/h*** | **1.004314456** | **0.99999c** | **223.6073568** |
| **2x10E+8 *km/h*** | **1.017600788** | **0.999999c** | **707.1067812** |
| **c/3** | **1.060660172** | **0.9999999c** | **2236.067978** |
| **c/2** | **1.154700538** | **0.99999999c** | **7071.067812** |
| **2c/3** | **1.341640786** | **0.999999999c** | **22360.67978** |
| **3c/4** | **1.511857892** | **c** | **∞** |

So, from Table 1 follows that for small velocities 50,000 *km*/*h* to 200,000 *km*/*h*, the absolute and the relative stress tensor are nearly the same. Furthermore, for bigger velocities like *c*/3, *c*/2 or 3*c*/4 (*c* = speed of light), the variable $\gamma$ takes values more than the unit and thus, relative stress

tensor is very different from the absolute one. In addition, for values of the velocity for the moving structure near the speed of light, the variable $\gamma$ takes bigger values, while when the velocity is equal to the speed of light, then $\gamma$ tends to the infinity.

Hence, the Singular Integral Operators Method (S.I.O.M.) as was proposed by E.G.Ladopoulos [4], [8], [9], [11], [12], [13], [15] and E.G.Ladopoulos et al. [22] will be used for the numerical evaluation of the stress tensor (3.11), for every specific case.

## 5. Conclusions

By the current research in the area of aerospace and aeronautical technologies the theory of *"Universal Mechanics"* has been introduced and applied for the design of the aircraft with speeds in the range of 50,000 km/h. Such a design and construction of the future aircraft will be applied to an increased market share of big Aeronautical Industries. In addition, *"Universal Mechanics"* has been applied for the design of the absolute spacecraft moving with very high speeds, even approaching the speed of light, as the plan of the big Space Agencies is to achieve such spacecraft in the future. The future investigation concerns to the determination of the proper composite materials or any other kind of materials for the construction of the future spacecraft, as usual composite solids are not suitable for such constructions.

The theory of *"Universal Mechanics"* and correspondingly the *"Universal Equation of Elasticity"* and the *"Universal Equation of Thermo-Elasticity"* show that there is a considerable difference between the absolute stress tensor of the airframe even in the range of speeds of 50,000 km/h. For bigger speeds the difference between the two stress tensors is very much increased. *"Universal Mechanics"* results as a combination of the theories of *"Relativistic Elasticity"* and *"Relativistic Thermo-Elasticity"*.

So, for the structural design of the future aircraft and spacecraft will be used the stress tensor of the airframe in combination to the singular integral equations. Such a stress tensor is reduced to the solution of a multidimensional singular integral equation and for its numerical evaluation will be used the Singular Integral Operators Method (S.I.O.M.). For the future aircraft and spacecraft of very high speeds special aerodynamics should be used.

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